Different ways of calculating relative returns

[Nematrian website page: RelativeReturnComputations, © Nematrian 2015]

Conventionally, a fund's return relative to a benchmark return is calculated as the arithmetic difference between the two (perhaps both annualised, following some specified annualisation convention), i.e. as $r_a = f - b$, where f is the fund return and b is the benchmark return. This is known as an *arithmetic relative return*.

Returns compound through time, and hence log returns add through time. It is therefore usually theoretically more appropriate to focus on *logarithmic relative returns*, i.e.:

$$r_l = \log(1 + f) - \log(1 + b)$$

Here, f and b need to be quoted as fractions, i.e. an arithmetic return of 10% is represented by 0.1, and it is conventional to use natural logarithms, so its corresponding log return is $\log(1 + 0.1) = \log(1.1) = \ln(1.1) = 0.09531$.

However, logarithms are not a universally well understood concept amongst the readers of performance reports, who do not necessarily have a mathematical or scientific background. A hybrid that is thus commonly used is to quote geometric relative returns, defined as:

$$r_g = \frac{1 + f}{1 + b}$$

Again, f and b need to be quoted as fractions.

The Nematrian performance measurement functions generally allow the user to choose between these different conventions, where relevant.

We may then define corresponding relative risk measures, such as ex ante tracking error (which is inherently a relative risk measure) and relative Value-at-Risk (i.e. relative VaR).