High resolution extended image near field optics

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These pages describe how in principle it is possible to create a near field optical microscope or lithographic device capable of creating an entire simultaneous extended image with a resolution similar to that achievable a point at a time by a scanning near field optical device. They also describe how in principle a similar resolution enhancement can be achieved with telescopes, although only at the expense of a loss of light collecting ability.

1. Introduction and Conclusions

[HighResOptics1]

How practical it is to achieve arbitrarily high resolution with a microscope, telescope or photolithographic device?

In days gone by, this topic would have had a clear answer. The wave-like nature of light places a limit on the effective resolving power of conventionally formulated (i.e. 'far field') devices. This limit is given by the well-known Rayleigh's equation, which states that the minimum distance able to be resolved by such a microscope or a photolithographic device, R, is set by:

$$R = \frac{k\lambda}{NA}$$

In this formula, λ is the wavelength of the light used, NA is the numerical aperture of the device and k is a constant that depends on the image formation technique used and (for microscopy or photolithography) the specific resist being used in the device. A similar equation, involving the aperture size and wavelength limits the angular resolving power of a (conventional) telescope.

However, given the huge technological and economic benefits accruing from squeezing ever finer circuitry onto microchips, cunning researchers have squeezed down the value of R using a variety of techniques. Some have involved improving k using 'unconventional' designs, some have involved shortening the wavelength of light being used, and some have involved lenses with enhanced numerical apertures, see e.g. Ito and Okazaki (2000) for further details.

Perhaps the most extreme example of an 'unconventional' design is the scanning near field optical microscope ('SNOM'). This works in a manner similar to a scanning tunnelling electron microscope, in that the microscope is placed a fraction of a wavelength away from the object being imaged. The Rayleigh equation no then longer applies and arbitrarily accurate resolution is in theory possible (even to the level of individual molecules or, with scanning tunnelling electron microscopes, individual atoms).

SNOM's operate by shining light through a very small hole which is placed very close to the object being imaged or projected onto. In principle, SNOM's have arbitrarily high resolving power, since in theory their accuracy is limited only by the size of the hole through which the light is shone. In practice, however, their resolution is limited because of the finite skin depth of real metals, or in other words because idealised metals with infinitely high (complex) refractive indices do not actually exist. This places a lower limit on the effective hole size that it is possible to achieve in practice (although it is possible circumvent this limit to some extent by arranging for the light source to be a single molecule).

It is possible to use the same approach in reverse for lithography, although Ito and Okazaki (2000) express the view that "although such approaches are useful for producing individual nano-structures for the investigation of nanometre-scale devices, the throughput is likely to always remain impracticably low for commercial application".

What is less clear is whether near field optical lithography (or microscopy) necessarily has to operate a point at a time, and therefore whether the assumed commercial barrier indicated by Ito and Okazaki (2000) is correct. We show that this assumption is probably false and that a resolving power similar to that achievable by existing SNOM designs should in principle be achievable by a near field device that creates an extended spatial image 'all at once'. We shall also describe how in principle this approach might be used to create a telescope that also circumvents the Rayleigh resolution limit, despite how far away any practical object being viewed will be from the viewing telescope.

Whether these ideas will ever prove practical and/or commercial is more difficult to say – they were originally set out in a patent application in 2001, see Kemp (2001) and have not developed further since then. Other arguably more commercial techniques have since been developed that also 'stretch the envelope' of what is possible with high resolution optical lithography. However, continued improvement in resolving power is one of the key drivers needed to maintain Moore's law and the microchip revolution we are all benefiting from. Perhaps, one day, the ideas set out in these pages will therefore prove useful and fruitful, if other methods of stretching the envelope run out of steam.

2. An idealised symmetric extended image near field imaging device

[HighResOptics2]

Consider a rotationally symmetric optical layout with an (axial) cross-section as per Figure 1. This consists of two large highly elongated truncated ellipsoidal mirrors, with plane mirrors (perpendicular to the axis of rotation) placed at the left and right hand ends of the arrangement. The centre of the mirror at the left hand end of the layout, A, is one of the focal points of the ellipsoid that forms the left half of the layout. The centre of the mirror at the right hand end of the arrangement, B, is one of the focal points of the ellipsoidal mirror that forms the right half of the layout. Both ellipsoidal mirrors also share a focal point at C, half-way along the layout (i.e. the two ellipsoidal mirrors are confocal). ACB forms a straight line, so the ellipsoidal mirrors are also coaxial.

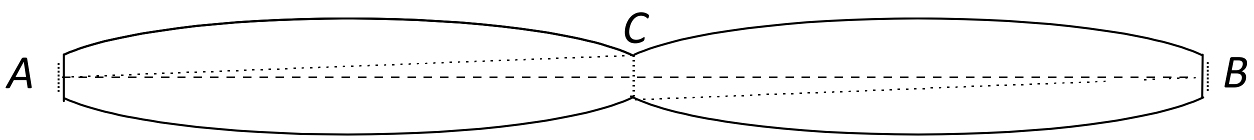

Figure 1: A large highly elongated symmetric truncated confocal and coaxial ellipsoidal mirror pair, with plane mirrors perpendicular to the axis of rotation at each end of the layout

Suppose that:

(a) The plane mirrors at each end of the layout are thin 'nearly fully silvered' idealised reflectors (with the silvering pointing inwards, i.e. in each case towards C), i.e. they transmit a small fraction of light incident on them but otherwise perfectly reflect all of the light incident onto them);

(b) Both ellipsoidal mirrors are 'partly silvered' idealised reflectors, i.e. perfectly reflect a proportion of all of the light incident onto them. It is assumed that behind them is a perfect absorber, so that any light transmitted through them can be ignored. The extent to which they need to be partially rather than fully silvered depends on the extent to which light that has bounced back and forth between the plane mirrors would corrupt the image formation. Image blurring arising because of these path trajectories can be eliminated by making the ellipsoidal mirrors only slightly mirrored, but at the expense of less light being available to create the image;

(c) All four mirrors are many wavelengths in size;

(d) The ellipsoidal mirrors are arbitrarily elongated (so the angle subtended by the hole at C on either A or B is arbitrarily small);

(e) A flat object is placed a small fraction of a wavelength to the left of A; and

(f) The object , whilst many wavelengths in size, is only an arbitrarily small fraction of the size of the entire aperture formed by the rim of the truncated ellipsoidal mirror (so is not drawn to scale in Figure 1).

What image of the object in (e) would be formed a small fraction of a wavelength to the right of B?

In the *absence* of the two end plane mirrors, the ellipsoidal mirror pair form an aplanatic layout, with object and image planes at A and B respectively. We would therefore expect it to create a clean, but Rayleigh resolution-limited, image at B of the object placed at A in a manner similar to any other 'conventional' imaging arrangement. For the image not to suffer material amounts of spherical aberration, we need the object to be small relative to the distance between the focal point and the nearest rim of the ellipsoidal mirror, but given design feature (e) the object could still be many wavelengths in size before this became an issue. Objects placed a sufficiently small fraction of a wavelength behind at A would therefore form an image a sufficiently small fraction of a wavelength behind B that is arbitrarily close in form to a conventional Rayleigh resolution-limited image.

However, there are three ways in which the complete layout described in this hypothetical situation differs from that a 'conventional' imaging arrangement:

(i) The nearly fully silvered plane mirror at the right hand end of the layout converts the device from a far-field to a *near-field* device. There is now an active part of the device near to, indeed exactly in the image plane;

(ii) The layout subtends a solid angle onto the image plane at B that is almost the maximum possible onto a plane. The only rays that are missing from the complete span of possible ray trajectories are ones that would otherwise have been coming from the vicinity of C. Design feature (d) means that these form an arbitrarily small proportion of the total angle span onto the image plane and so in the limit can be ignored; and

(iii) The nearly fully silvered plane mirror at the left hand end of the layout constrains the nature of the light waves entering the cavity formed by the ellipsoidal mirrors, and thus also constrains the nature of the waves converging onto the image plane.

Our assertion is that inclusion of these non-standard aspects to the layout result in an image of A being formed at B that is **no longer subject to the Rayleigh resolution limit**. Indeed the image should be arbitrarily accurate, to the extent that it is possible to create such an idealised layout in practice. Moreover, if the plane mirrors are sufficiently close to being fully silvered as per design feature (a), then the device would create an *extended* image that circumvents the Rayleigh resolution limits that might make SNOM-type technology more commercially viable.

Readers may, however, object that, even if this assertion were true, the design features needed for the above device to work would involve some carefully crafted limiting properties. Some of these relate to the physical characteristics of the materials used to make the mirrors, some relate to the dimensions of the layout (both width and length) relative to the object being imaged and some relate to the proportion of light leaving the object that is reconstituted to form the image. Moreover, with the above design the object and image are of the same size, limiting the practical usefulness of the proposed design for microscopy and certainly rendering it useless for telescopy.

The main aim of discussing the above layout is thus to elucidate the principles involved and to suggest ways in which the layout would need to be refined were it to be applied in practice.

3. Exact radiating solutions to Maxwell's equations in a vacuum

[HighResOptics3]

The explanation of the unusual properties of the idealised optical layout described in Section 2 lies in the behaviour of certain types of exact solutions of Maxwell's equations in the presence of idealised plane mirrors.

Before exploring these further, let us first the nature of radiating solutions to Maxwell's equations *in a vacuum*. These can be written as superpositions of (potentially infinitely many) outwardly and inwardly radiating electric and magnetic dipoles.

Born & Wolf (1980) describe the behaviour of a single *outwardly* radiating *electric* dipole, characterised by source location $\boldsymbol{a}$, a unit vector, $\boldsymbol{n}$, describing the direction in which the dipole is pointing, and an electric polarization vector $\boldsymbol{P}$ whose value at point $\boldsymbol{r}$ and at time $\boldsymbol{t}$ is given by $\boldsymbol{P}(\boldsymbol{r},t) = p(t)\delta(\boldsymbol{r}-\boldsymbol{a})\boldsymbol{n}$, where δ is the Dirac function and p is a function of time. The full (i.e. exact) solution to Maxwell's equations (in a vacuum) for such a dipole then has the following form, where $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{H}$ and $\boldsymbol{N}$ are the electric field, electric displacement, magnetic and magnetic induction vectors respectively:

$$\boldsymbol{B} = \boldsymbol{H} = \left\{ \frac{[\dot{p}]}{cR^3} + \frac{[\ddot{p}]}{c^2R^2} \right\} \boldsymbol{n} \times (\boldsymbol{r} - \boldsymbol{a})$$

$$\boldsymbol{E} = \boldsymbol{D} = \left\{ \frac{3[p]}{R^5} + \frac{3[\dot{p}]}{cR^4} + \frac{[\ddot{p}]}{c^2R^3} \right\} \big(\boldsymbol{n}.(\boldsymbol{r} - \boldsymbol{a})\big)(\boldsymbol{r} - \boldsymbol{a}) - \left\{ \frac{[p]}{R^3} + \frac{[\dot{p}]}{cR^2} + \frac{[\ddot{p}]}{c^2R} \right\} \boldsymbol{n}$$

Here $R = |\boldsymbol{r} - \boldsymbol{a}|$ and c is the speed of light. Square brackets denote retarded values, i.e. $[f] = f(t - R/c)$.

The form of this solution is slightly easier to visualise in spherical polar coordinates (R, θ, ϕ) taking the origin as the source location, $\boldsymbol{a}$, θ as the angle between $\boldsymbol{n}$ and $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{a}$ and ϕ as the angle that the projection of $\boldsymbol{R}$ onto the plane perpendicular to $\boldsymbol{n}$ makes with a constant vector perpendicular to $\boldsymbol{n}$. If $\boldsymbol{i}_R$, $\boldsymbol{i}_\theta$ and $\boldsymbol{i}_\phi$ are unit vectors in the direction of increasing R, θ and ϕ respectively then the outwardly radiating electric dipole has the form $\boldsymbol{E} = E_R \boldsymbol{i}_R + E_\theta \boldsymbol{i}_\theta$ and $\boldsymbol{H} = H_\phi \boldsymbol{i}_\phi$ where:

$$E_R = 2\left(\frac{[p]}{R^3} + \frac{[\dot{p}]}{cR^2} \right) \cos\theta$$

$$E_\theta = \left(\frac{[p]}{R^3} + \frac{[\dot{p}]}{cR^2} + \frac{[\ddot{p}]}{c^2R} \right) \sin\theta$$

$$H_\phi = \left(\frac{[\dot{p}]}{cR^2} + \frac{[\ddot{p}]}{c^2R} \right) \sin\theta$$

The form of the *inwardly* radiating *electric* dipole, i.e. the time reversed solution, can be found by replacing t by $-t$ and $[\dot{p}]$ by $-[\dot{p}]$ (since $d/d(-t) = -d/dt$) and by placing a negative sign in front of the corresponding expressions for B and H (since $\dot{\boldsymbol{B}} = -c.curl(\boldsymbol{E})$).

The corresponding outwardly and inwardly radiating *magnetic* dipoles have $\boldsymbol{E}$ replaced by $\boldsymbol{B}$ and $\boldsymbol{B}$ replaced by $-\boldsymbol{E}$, given the symmetric nature of Maxwell's equations in a vacuum. For reasons that will become obvious later on, we will concentrate on these latter types of dipoles in the remainder of this analysis.

We can further decompose each of these dipoles into superpositions of sinusoidally time-varying dipoles all with the same origin, using Fourier analysis. These will be the types of dipoles that we will concentrate on in the remainder of this analysis. For *magnetic* dipoles with $p(t) = Re\big(qe^{-iwt}\big)$, q and w constant, these have the following form (where $Re(z)$ is the real part of the complex number z and i is the square root of -1):

<u>Outwardly radiating (magnetic) dipoles</u>

$$\boldsymbol{E}(\boldsymbol{r}, t) = Re\big(\boldsymbol{E}(\boldsymbol{r}, t; \boldsymbol{a}, \boldsymbol{n}, q, w, +)\big)$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = Re\big(\boldsymbol{B}(\boldsymbol{r}, t; \boldsymbol{a}, \boldsymbol{n}, q, w, +)\big)$$

<u>Inwardly radiating (magnetic) dipoles</u>

$$\boldsymbol{E}(\boldsymbol{r}, t) = Re\big(\boldsymbol{E}(\boldsymbol{r}, t; \boldsymbol{a}, \boldsymbol{n}, q, w, -)\big)$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = Re\big(\boldsymbol{B}(\boldsymbol{r}, t; \boldsymbol{a}, \boldsymbol{n}, q, w, -)\big)$$

where

$$\boldsymbol{E}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,+) = qe^{-iwt}\boldsymbol{F}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)$$
$$\boldsymbol{B}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,+) = qe^{-iwt}\boldsymbol{G}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)$$
$$\boldsymbol{E}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,-) = qe^{iwt}\boldsymbol{F}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)$$
$$\boldsymbol{B}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,-) = qe^{iwt}\boldsymbol{G}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)$$
$$\boldsymbol{F}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w) = \frac{e^{iwR/c}}{R}\left(\frac{iw}{cR}+\frac{w^2}{c^2}\right)\boldsymbol{n}\times\boldsymbol{h}$$
$$\boldsymbol{G}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w) = \frac{e^{iwR/c}}{R}\left(\left(\frac{3}{R^2}-\frac{3iw}{cR}-\frac{w^2}{c^2}\right)(\boldsymbol{n}.\boldsymbol{h})\boldsymbol{h}-\left(\frac{1}{R^2}-\frac{iw}{cR}-\frac{w^2}{c^2}\right)\boldsymbol{n}\right)$$
$$R = |\boldsymbol{r}-\boldsymbol{a}|$$
$$\boldsymbol{h} = \frac{\boldsymbol{r}-\boldsymbol{a}}{R}$$

4. Exact radiating solutions to Maxwell's equations in the presence of idealised plane mirrors

[HighResOptics4]

Consider now the behaviour of inwardly and outwardly radiating (magnetic) dipoles in the presence of an idealised plane mirror, i.e. the solution, say, in the half space $z \geq 0$ arising from a dipole whose origin in Cartesian coordinates (x, y, z) is given by $\boldsymbol{a} = (a_1, a_2, a_3)$ $(a_3 \geq 0)$ and whose direction is given by $\boldsymbol{n} = (n_1, n_2, n_3)$ if there is:

(a) A vacuum in the region $z \geq 0$; and
(b) A perfectly conducting plane mirror at $z = 0$.

As Born & Wolf (1980) explain, the exact boundary condition satisfied on the plane $z = 0$ is that the component of $\boldsymbol{E}$ tangential to $z = 0$ is zero.

Now let $\boldsymbol{b} = (a_1, a_2, -a_3)$ and $\boldsymbol{m} = (n_1, n_2, -n_3)$. The reason we focus on *magnetic* rather than *electric* dipoles using the terminology in Section 3 is that the superposition of two such equal magnitude and in-phase dipoles, one emanating at a pointing in direction n and the other emanating at b and pointing in the direction m then *exactly* satisfies the required boundary condition at $z = 0$. Suppose we write this superposition as:

$$\boldsymbol{E}_{\pm}(\boldsymbol{r},t) = Re\big(\boldsymbol{E}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,\pm)+\boldsymbol{E}(\boldsymbol{r},t;\boldsymbol{b},\boldsymbol{n},q,w,\pm)\big)$$
$$\boldsymbol{B}_{\pm}(\boldsymbol{r},t) = Re\big(\boldsymbol{B}(\boldsymbol{r},t;\boldsymbol{a},\boldsymbol{n},q,w,\pm)+\boldsymbol{B}(\boldsymbol{r},t;\boldsymbol{b},\boldsymbol{n},q,w,\pm)\big)$$

It exactly satisfies the boundary condition because at $z = 0$ we have $\boldsymbol{r}-\boldsymbol{a} = \boldsymbol{r}-\boldsymbol{b}$ and $\boldsymbol{n}\times(\boldsymbol{r}-\boldsymbol{a})+\boldsymbol{m}\times(\boldsymbol{r}-\boldsymbol{b}) = \big(0,0,2n_1(r_2-a_2)-2n_2(r_1-a_1)\big)$, if $\boldsymbol{r} = (r_1, r_2, r_3)$ in Cartesian coordinates. So the x and y components of the electric field at $z = 0$ are both zero and $\boldsymbol{E}_{\pm}$ is thus exactly perpendicular to the mirror.

Consider further the special case of the above where $a_3 = 0$ and $n_3 = 0$. We then have $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{n} = \boldsymbol{m}$, the dipole is emanating from the plane mirror itself and the solutions take the form:

$$\boldsymbol{E}_{\pm}(\boldsymbol{r},t) = Re\left(2qe^{\mp iwt}\boldsymbol{F}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)\right)$$
$$\boldsymbol{B}_{\pm}(\boldsymbol{r},t) = Re\left(2qe^{\mp iwt}curl\,\boldsymbol{F}(\boldsymbol{r};\boldsymbol{a},\boldsymbol{n},w)\right)$$

Surfaces of constant phase for this special case are hemispheres centred about $\boldsymbol{a}$. The direction and amplitude of the real physical values of $\boldsymbol{E}$ on each such hemisphere then have the form $C(R,t)\boldsymbol{n} \times \boldsymbol{h}$, i.e. $\boldsymbol{E}$ is perpendicular to both the direction of the corresponding radius vector and the direction of the dipole and has a maximum amplitude proportional to the sine of the angle between these two vectors.

Consider also the situation where we have the special case solution form as above *and* we place a perfectly conducting metallic hemispherical mirror placed at $|\boldsymbol{r} - \boldsymbol{a}| = S$ (in the region $z \geq 0$) for some constant S. As $\boldsymbol{E}$ is exactly tangential to each such hemisphere, any *exact* outwardly radiating (magnetic) dipole from a will strike the hemisphere, be reflected with a 180 degree phase transition and create exactly the right boundary conditions to create an exact inwardly radiating (magnetic dipole).

If the hemisphere was centred at $\overline{\boldsymbol{a}}$, some point on the plane mirror not far from $\boldsymbol{a}$, then outwardly radiating dipoles from $\boldsymbol{a}$ would not have the right characteristics to generate the exact boundary conditions needed for an equivalent inwardly radiating dipole, at least not one that radiates back to $\boldsymbol{a}$. However, any dipole emanating from $\boldsymbol{a}$ that bounced a second time off the plane mirror and then of the hemispherical mirror would then have the right characteristics, to first order, to create the required boundary conditions. So, if S is sufficiently large compared to $|\overline{\boldsymbol{a}} - \boldsymbol{a}|$ then the layout would again create an arbitrarily accurate inwardly radiating (magnetic) dipole with destination $\boldsymbol{a}$.

5. Creating arbitrarily accurate images

[HighResOptics5]

Why have we analysed the relatively contrived optical layout described in Section 4? We cannot in practice create mirrors with this level of perfection, and even if we could the light would bounce arbitrarily often between, $\boldsymbol{a}$, the source (destination) of the outwardly (inwardly) radiating (magnetic) dipole and the hemispherical mirror placed at $|\boldsymbol{r} - \boldsymbol{a}| = S$.

The point is that the optical layout we have contrived has an unusual and potentially desirable feature. The light coming back towards the plane mirror at at $z = 0$ is entirely concentrated onto $\boldsymbol{a}$. In short, we have created a device that *arbitrarily accurately focuses* light onto a, even if we needed a rather contrived way of generating the relevant oncoming wavefronts to do so.

The key additional insight is to realise that the elongated confocal and coaxial optical layout described at the start of Section 2 has 'essentially' the same optical characteristics as going two times round the the hemispherical and plane mirror optical layout described in Section 4. By 'essentially' the same we mean that if the ellipsoids in Section 2 were elongated enough (so that the hole near their joint axis subtended an arbitrarily small solid angle on each end mirror) and large enough relative to the size of the image at roughly the centre of the image plane then the optical characteristics of exhibited by the layout in the vicinity of the centre of the object plane would be the same as that created by the layout in Section 4.

Any outwardly radiating (magnetic) dipole coming from a given point in the object plane near its centre is thus converted to an inwardly radiating (magnetic) dipole that concentrates onto the corresponding point in the image plane, completely circumventing the Rayleigh resolution limit. This is the case for *all* points sufficiently near the centre of the object/image plane, and hence involves an entire extended image, as long as design feature (f) of Section 2 applies, and as long as the mirror in the image plane is sufficiently well mirrored that as far as any individual incoming dipole is concerned, the plane mirror is essentially fully mirrored (hence the need for design feature (a) of Section 2).

6. Understanding how the layout can circumvent the Rayleigh resolution criterion

[HighResOptics6]

Those in the field of optics who have grown up being taught the Rayleigh resolution criterion may take some convincing that such a device really would circumvent it, even though at no point did our argument introduce the wavelength of the light being focused (except implicitly in the sizes of some parts of the layout). The key points to note are:

- The device we have described is an aplanatic optical layout, so would produce an arbitrarily accurate image if the Rayleigh resolution criterion didn't apply.

- The device is an extreme example of a 'near field' device, by which we mean a layout with an active component only fraction of a wavelength from the image. The introduction of the plane mirror positioned *at* the image plane makes it 'near field'. Indeed, we see that it is precisely *because* there is such a mirror there that any inwardly radiating dipole centred on the image plane continues to increase in magnitude as we approach closer and closer to the dipole centre. Without such a mirror, the light waves would in effect refract/diffract away via the 'gap' in the boundary conditions created by the missing plane mirror. It is the *lack* of such a mirror (or equivalent optical element creating equivalent boundary conditions) that makes a device not 'near field' and hence 'far field'.

Some might also argue that circumventing the Rayleigh resolution criterion in the manner being proposed is intrinsically objectionable from the perspective of quantum mechanics, given Heisenberg's uncertainty principle. The argument would be that it 'ought' not to be possible to create an arbitrarily accurate image in this manner because doing seems to provide us with a way of simultaneously achieving an arbitrarily accurate measurement of the location of a light wave and of its momentum (given knowledge of the frequency of the light being used for imaging purposes).

To solution to this quantum mechanical paradox is to note that the device only transmits a fraction of the light incident on the image plane through to the image detector. The greater its accuracy the more light it 'rejects'. It therefore corresponds to an example of 'weak measurement' as per Aharonov et al (1988) or Starling et al. (2009) as reported in Steinberg (2010). The greater its accuracy, the more it relies on 'weak measurement' as a means of apparently circumventing the Heisenberg uncertainty principle, i.e. the more photons it needs to use to achieve the required accuracy.

7. Further comments

[HighResOptics7]

For photolithography and perhaps also for some types of microscopy, having an image the same size as the object is not necessarily a fundamental problem. However for telescopy and most types of microscopy it is. Objects we are interested in viewing through telescopes are typically large and far away, whilst objects we are interested in viewing through microscopes may not be far away but usually we want to incorporate some magnification into the process.

It is possible to create aplanatic analogues of double confocal and coaxial ellipsoids introduced in Section 2 that still span the complete range of angles onto a plane, and can thus still in principle make use of nearly fully silvered plane mirrors to support imploding dipole wavefronts. Indeed, the main reason I was led to explore the exact behaviour of solutions to Maxwell's equations was because I had been considering the possible use of such mirror layouts for solar power concentration and solar powered flight purposes.

However:

(a) Whilst equivalent aplanatic layouts that involve magnification can be identified, they do not appear by themselves to create the required boundary conditions to generate perfectly imploding dipole solutions. Instead it seems to be necessary to rotate (and attenuate) by different amounts the light falling on different parts of the telescope or microscope aperture in order to achieve the desired boundary conditions.

    Such an analysis highlight that is the presence of the plane mirror *at the image plane* which is of particular importance in achieving superresolution, as we might have surmised from the discussion in Section 6 about how and why this sort of superresolution does not contradict established physical principles.

(b) For astronomical telescopes, it does in principle appear to be possible to achieve a resolution better than that implied by the Rayleigh resolution criterion, by linking two such devices as in (a) of different sizes back-to-back to achieve a suitable level of magnification. However, it is doubtful whether such a device would be as effective as one that involved multiple individual telescopes positioned some way away from each other, which is a well-established technique for boosting resolving power. In particular, we noted in Section 6 that the improvement in accuracy arises principally because we are discarding most of the light falling on the layout, impairing such a telescope's light gathering ability.

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[HighResOpticsRefs]

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