# A potential way in which the Central Limit Theorem (CLT) can break down via combinations of lots of independent gamma distributed random variables

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The Gamma distribution has the the 'summation' property that if $X_i \sim \Gamma(k_i, \theta)$ for $i = 1, \dots, n$ and the $X_i$ are independent then $\sum_{i=1}^{n} X_i \sim \Gamma(\sum_{i=1}^{n} k_i, \theta)$.

So, suppose $X_i \sim \Gamma(k/n, \theta)$. Then $\sum_{i=1}^{n} X_i \sim \Gamma(k, \theta)$. Thus we appear to be combining more and more (independent) random variables each of which has smaller and smaller mean, $(k/n)\theta$, and variance, $(k/n)\theta^2$, so we might expect the Central Limit Theorem (CLT) to apply.

In fact, for the CLT to apply we need somewhat more onerous regularity conditions to be satisfied, including a focus on $(1/n)\sum_{i=1}^{n} X_i$ and (usually) that the distributions of the $X_i$ do not change as $n$ changes (as well as being of finite variance). The above example does not satisfy these amplified regularity conditions because as $n$ changes the distribution of each $X_i$ changes. Although the means (and variances) get smaller and smaller (which you would have thought would help with satisfying the CLT), each individual $X_i$ becomes more and more skewed and has a greater and greater (excess) kurtosis, see e.g. the Nematrian webpage on the Gamma distribution.