

Formulae for prices and Greeks for European binary puts in a Black-Scholes world

[Nematrian website page: [BlackScholesGreeksBinaryPuts](#), © Nematrian 2015]

See [Black Scholes Greeks](#) for notation.

Payoff, see [MnBSBinaryPutPayoff](#)

$$Payoff = \begin{cases} 1 & \text{if } S < K \\ 0 & \text{if } S \geq K \end{cases}$$

Price (value), see [MnBSBinaryPutPrice](#)

$$Price = V = e^{-r(T-t)} N(-d_2)$$

Delta (sensitivity to underlying), see [MnBSBinaryPutDelta](#)

$$Delta = \frac{\partial V}{\partial S} = -\frac{e^{-r(T-t)} N'(-d_2)}{S\sigma\sqrt{T-t}}$$

Gamma (sensitivity of delta to underlying), see [MnBSBinaryPutGamma](#)

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{e^{-r(T-t)} d_1 N'(-d_2)}{S^2 \sigma^2 (T-t)}$$

Speed (sensitivity of gamma to underlying), see [MnBSBinaryPutSpeed](#)

$$Speed = \frac{\partial^3 V}{\partial S^3} = \frac{e^{-r(T-t)} N'(-d_2)}{\sigma^2 S^3 (T-t)} \left(-2d_1 + \frac{1 - d_1 d_2}{\sigma\sqrt{T-t}} \right)$$

Theta (sensitivity to time), see [MnBSBinaryPutTheta](#)

$$Theta = \frac{\partial V}{\partial t} = r e^{-r(T-t)} N(-d_2) - e^{-r(T-t)} N'(-d_2) \left(\frac{d_1}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}} \right)$$

Charm (sensitivity of delta to time), see [MnBSBinaryPutCharm](#)

$$Charm = \frac{\partial^2 V}{\partial S \partial t} = -\frac{e^{-r(T-t)} N'(-d_2)}{S\sigma\sqrt{T-t}} \left(r + \frac{1 - d_1 d_2}{2(T-t)} + \frac{d_2(r-q)}{\sigma\sqrt{T-t}} \right)$$

Colour (sensitivity of gamma to time), see [MnBSBinaryPutColour](#)

$$Colour = \frac{\partial^3 V}{\partial S^2 \partial t} = \frac{e^{-r(T-t)} N'(-d_2)}{\sigma^2 S^2 (T-t)} \left(r d_1 + \frac{2d_1 + d_2}{2(T-t)} - \frac{(r-q)}{\sigma\sqrt{T-t}} - d_1 d_2 \left(\frac{d_1}{2(T-t)} - \frac{r-q}{\sigma\sqrt{T-t}} \right) \right)$$

Rho(interest) (sensitivity to interest rate), see [MnBSBinaryPutRhoInterest](#)

$$Rho(Interest) = \frac{\partial V}{\partial r} = -(T-t) e^{-r(T-t)} N(-d_2) - \frac{\sqrt{T-t}}{\sigma} e^{-r(T-t)} N'(-d_2)$$

Rho(dividend) (sensitivity to dividend yield), see [MnBSBinaryPutRhoDividend](#)

$$Rho(Dividend) = \frac{\partial V}{\partial q} = (T - t)e^{-r(T-t)} N(-d_2)$$

Vega (sensitivity to volatility), see [MnBSBinaryPutVega](#)*

$$Vega = \frac{\partial V}{\partial \sigma} = e^{-r(T-t)} N'(-d_2) \frac{d_1}{\sigma}$$

Vanna (sensitivity of delta to volatility), see [MnBSBinaryPutVanna](#)*

$$Vanna = \frac{\partial^2 V}{\partial S \partial \sigma} = \frac{e^{-r(T-t)} N'(-d_2) (1 - d_1 d_2)}{S \sigma^2 \sqrt{T-t}}$$

Volga (or Vomma) (sensitivity of vega to volatility), see [MnBSBinaryPutVolga](#)*

$$Volga = \frac{\partial^2 V}{\partial \sigma^2} = \frac{e^{-r(T-t)} N'(-d_2) (d_1^2 d_2 - d_1 - d_2)}{\sigma^2}$$

* Greeks like vega, vanna and Volga/vomma that involve partial differentials with respect to σ are in some sense 'invalid' in the context of Black-Scholes, since in its derivation we assume that σ is constant. We might interpret them as applying to a model in which σ was slightly variable but otherwise was close to constant for all S , t etc.. Vega, for example, would then measure the sensitivity to changes in the mean level of σ . For some types of derivatives, e.g. binary puts and calls, it can be difficult to interpret how these particular sensitivities should be understood.