Techniques for Handling Extreme Events in the Context of Portfolio Construction

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Background

- Taking due account of the possibility of extreme events occurring is important but also challenging for many market professionals
  - *Insurers: Solvency II*. Mandates 1 in 200 year VaR, but we do not have 200 years of relevant historical data
  - *IORPs: Holistic Balance Sheets*. Can depend heavily on hopefully rare extreme credit events, e.g. a sponsor or a national pension protection scheme defaulting
  - *Banks: E.g. operational risk management*: many recent losses much larger than previously modelled (effective) upper limits
- Allowing for them in portfolio construction is particularly challenging
  - Need to balance *risk* versus *reward*, making it important to understand causes of extreme events and to avoid giving them *too much* emphasis

Agenda

- Why are return series often ‘fat tailed’?
- Extreme Value Theory (EVT) and possible refinements
- Modelling joint fat-tailed behaviour
- Lessons for portfolio construction

- See also Kemp, M.H.D. (2010) *Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails*. John Wiley & Sons and toolkit etc. at www.nematrian.com

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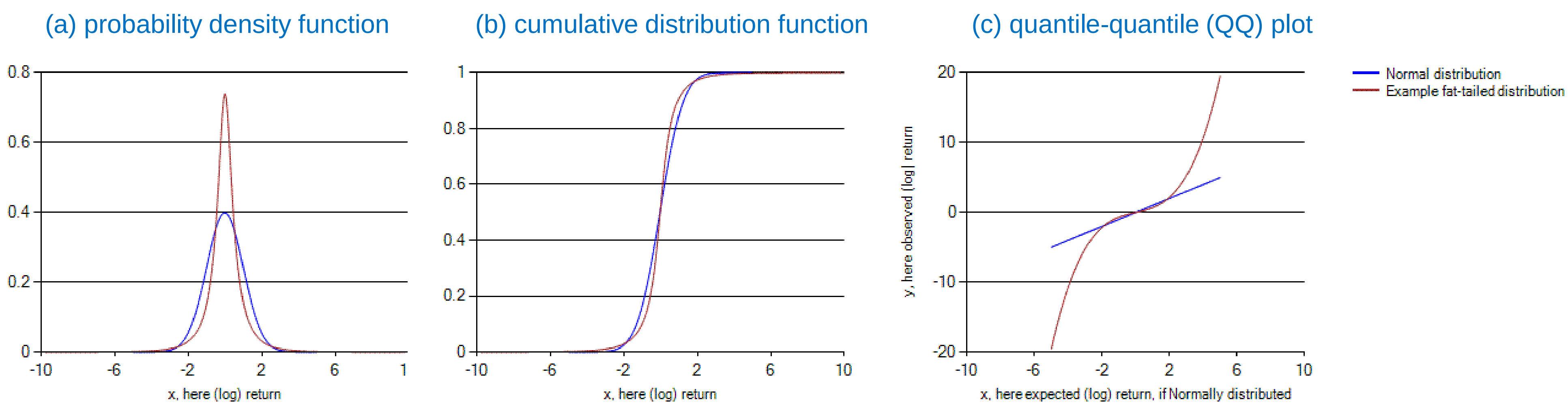
Modelling fat tails for *individual* risks

- 'Fat-tailed' means probability of extreme-sized outcomes seems to be higher than if coming from (usually) a (log) Normal distribution
- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below, i.e. QQ-plots
- Note: portfolio construction usually involves *multiple* assets / risk exposures

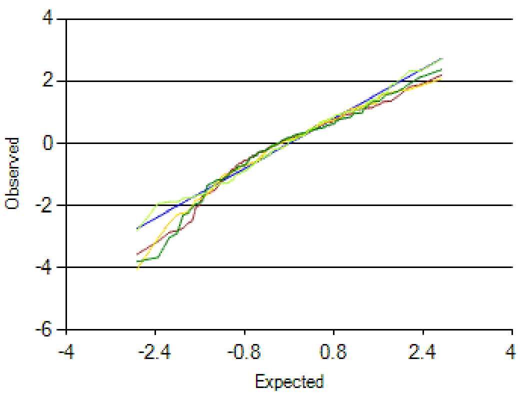


Source: www.nematrian.com

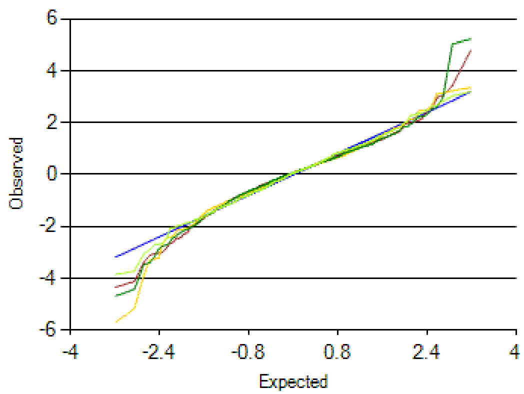
Many (most?) investment return series are ‘fat-tailed’

- Some instrument types intrinsically skewed (e.g. high-grade bonds, options)
- Others (e.g. equities) still exhibit fat-tails, particularly higher frequency data

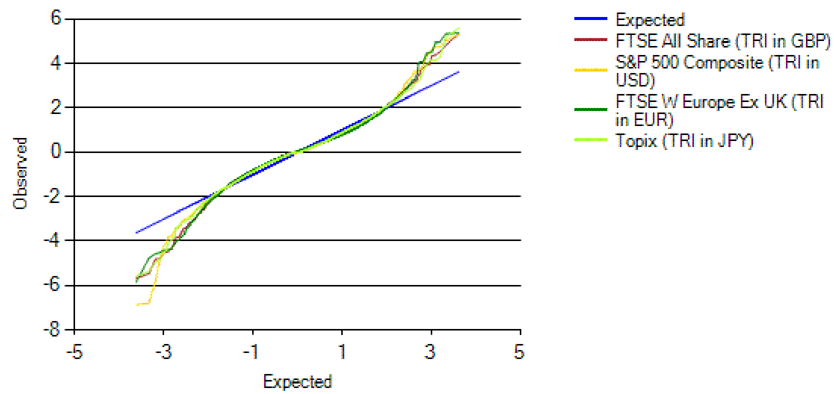
(1) Monthly returns



(2) Weekly returns



(3) Daily returns



Source: www.nematrian.com, Threadneedle, S&P, FTSE, Thomson Datastream

Returns from end June 1994 to end Dec 2007, charts show standardised logged returns

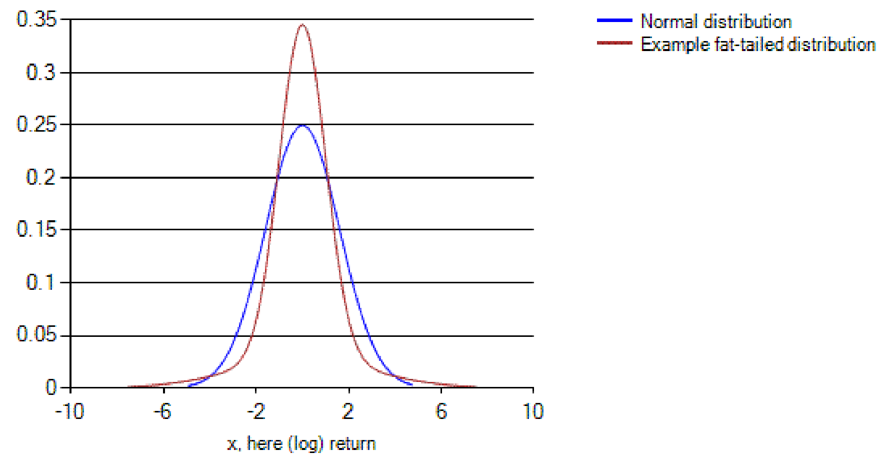
Nematrian

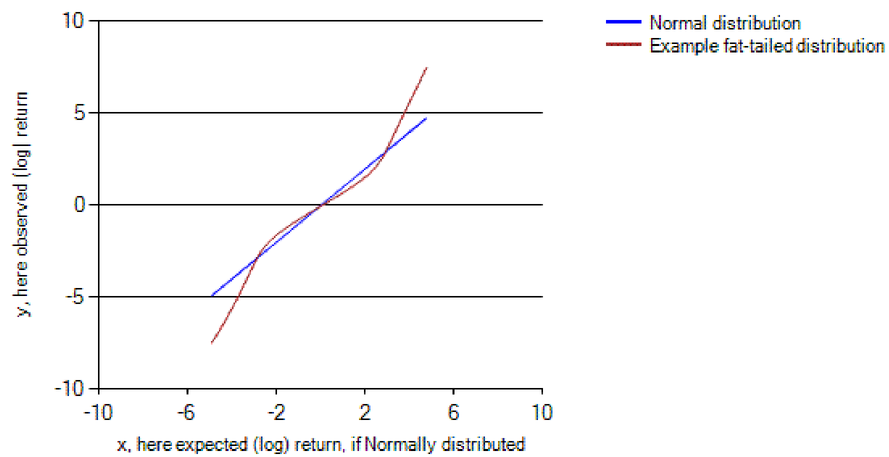
Why are return series often fat-tailed?

- As noted above, some instruments have intrinsically skewed behaviour
- More generally, an important source of fat-tailed behaviour is the *time-varying* nature of the world in which we live
  - Market / sector / instrument volatilities (and maybe other distributional characteristics) change through time
- Other sources include *crowded trades*, *leverage* and other *selection effects* such as manager behaviour being (consciously or unconsciously) biased towards strategies that are prone to fat-tailed behaviour

Distributional mixtures of Normal distributions

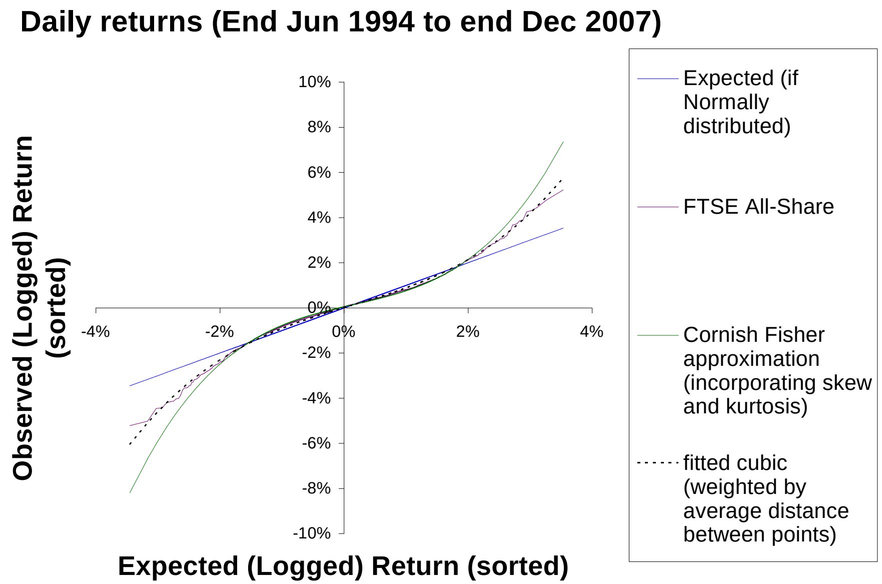
- E.g. draw X with prob p from N_1 and prob (1-p) from N_2
  - Quite different behaviour to *linear combination mixtures*, i.e. $a.X_1 + b.X_2$
- If N_1 and N_2 have same mean but different s.d.'s then distributional mixture is fat-tailed (if $p \neq 0$ or 1) but linear combination mixture isn't.
- Time-varying volatility is similar, involves draws from different distributions at different times



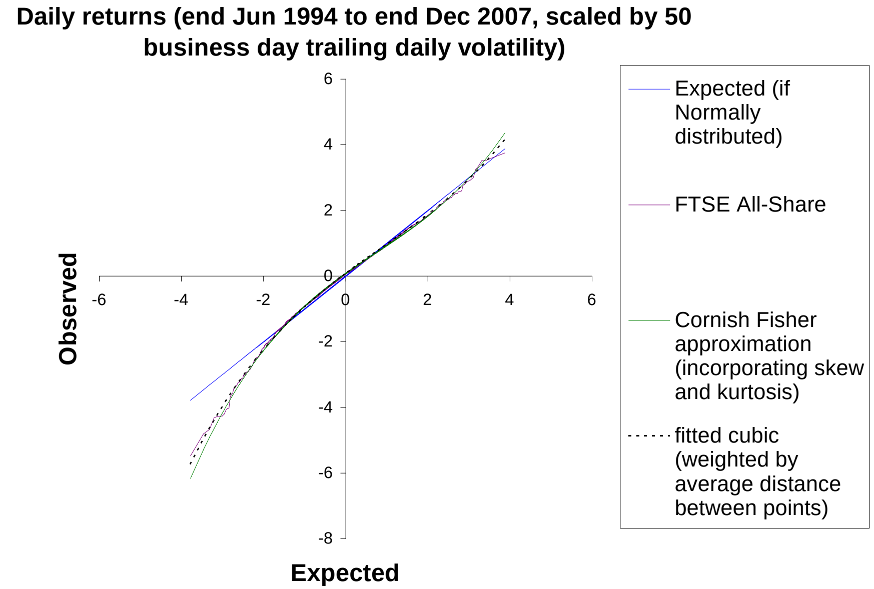


Impact of time-varying volatility

Raw Data



With Short-term Volatility Adjustment



Average extent to which tail exceeds expected level (average of 6 most extreme outcomes)				
	Downside (%)		Upside (%)	
	Unadj	Adj for vol	Unadj	Adj for vol
FTSE All-Share (in GBP)	54	41	42	3
S&P 500 (in USD)	68	70	50	7
FTSE Eur ex UK (in EUR)	48	53	54	-3
Topix (in JPY)	54	72	42	39

Source:

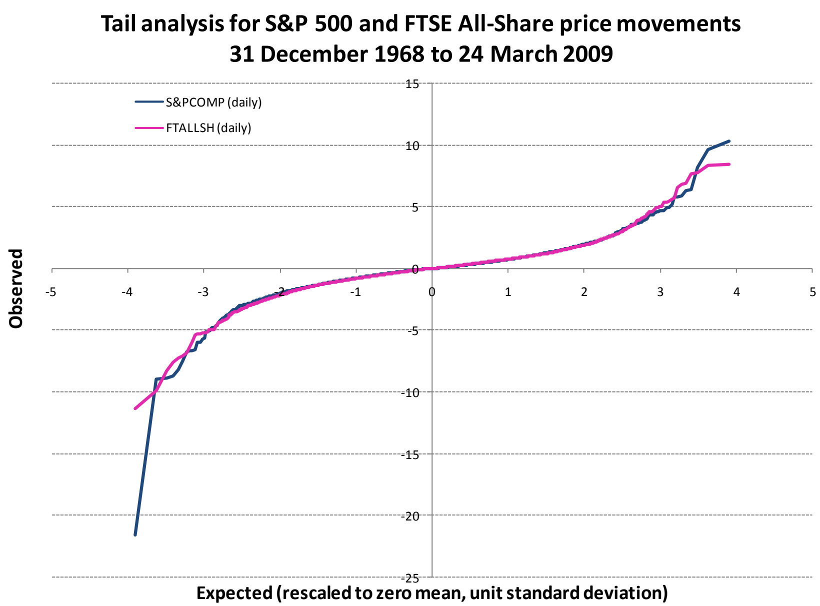
Threadneedle, FTSE

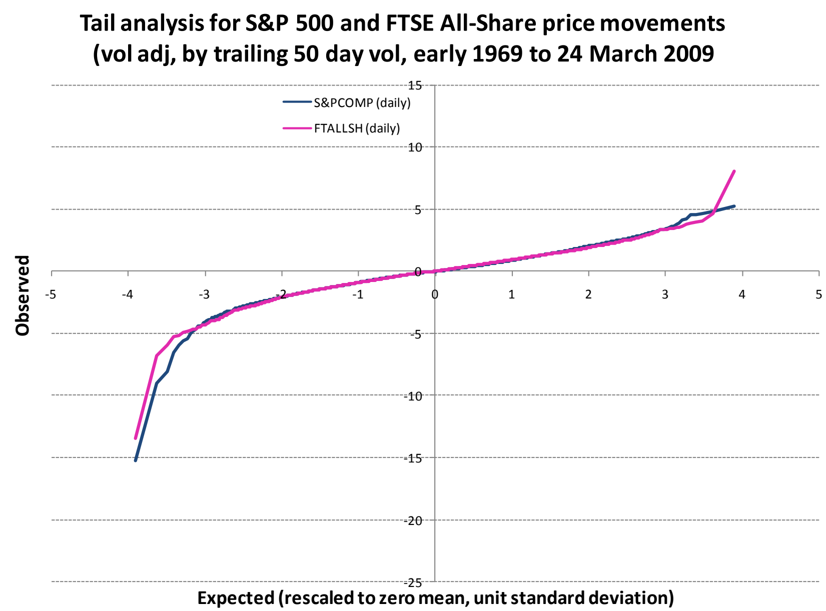
Thomson Datastream

Impact of time-varying volatility: longer-term data

Raw Data

With Short-term Volatility Adjustment





Source: Threadneedle, S&P, FTSE, Thomson Datastream

Crowded trades and leverage

- However, time-varying volatility does not explain all fat-tailed behaviour
- Some fat tails still seem to come “out of the blue”
  - E.g. Quant funds in August 2007 (marking the ‘start’ of the 2007-09 Credit Crisis?)
  - Too many investors in the same *crowded trades*?
- System-wide equivalents via leverage?
  - Leverage introduces/magnifies *liquidity* risk, *forced unwind* risk and *variable borrow cost* risk. May show up in an apparent shift in price basis
- Both involve behavioural finance effects

Selection effects (1)

- Selection effects (involving non-random choices by market participants) are important behavioural aspects of many actuarial problems
  - E.g. people who buy annuities tend to be healthier than the average of the population as a whole
- And could be important for portfolio construction too
  - Kemp (2010) explores what would happen if portfolio managers consciously (or unconsciously) selected equity industry mixes that in combination exhibited high kurtosis (e.g. because quantitative 'signals' underlying such strategies might stand out because they appeared non-random)
  - 1 in 200 Value-at-Risk (VaR) measures derived purely from volatilities of underlying factors on average c. 4 times too low for such 'selected' portfolios

Selection effects (2)

- Banks that failed during 2007-09 Credit Crisis were disproportionately biased towards strategies that depended on continuing favourable liquidity conditions
- Liquidity risk is highly fat-tailed
- So these banks were (consciously or unconsciously) biasing their business strategies towards ones that had high kurtosis

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Extreme Value Theory (EVT)

- EVT an enticing prospect
  - Appears to offer a mathematically sound way of identifying shape of the ‘tail’ of a distribution, and hence identifying likelihood of extreme (i.e. rare) events
  - Capital adequacy seeks to protect against (we hope) relatively rare events
  - Insurance and credit risk pricing can be dominated by potential magnitude and likelihood of large losses
- But bear in mind
  - Inherent unreliability of extrapolation, including into tail of a probability distribution
  - Possibility (indeed probability) that the world is not time stationary
  - Portfolio construction is inherently multivariate, i.e. choosing between different alternatives

Restatement of EVT results

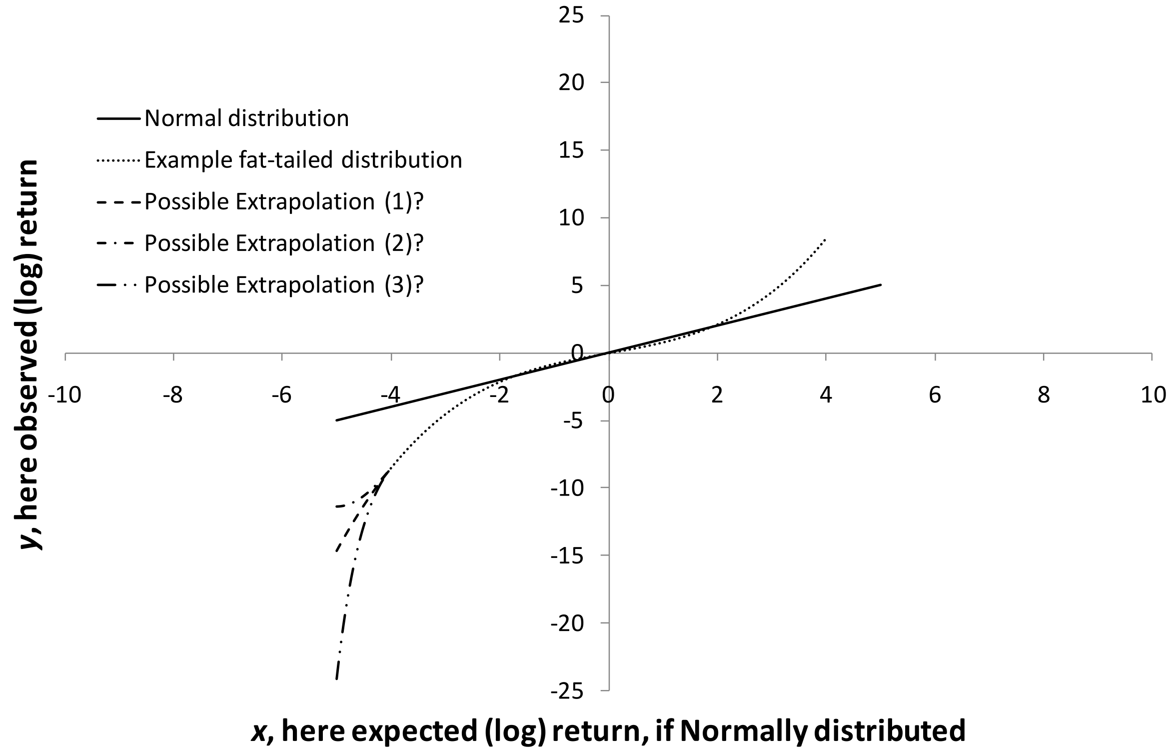
- Suppose interested in risk measures relating to losses, x_j
- EVT aims to supply two closely related results:

1. *Less relevant to risk management*: Distribution of 'block maxima' (or 'block minima'), i.e. maximum value of x_i in blocks of m observations of x, tends to a Generalised Extreme Value (GEV) distribution
2. *More relevant to risk management*: Distribution of 'threshold exceedances' (i.e. 'peaks-over-thresholds') tends to a generalised Pareto distribution (GPD) . Here u is a predetermined high threshold and we focus on realisations of x_j that exceed u, i.e.:

$$y_i \equiv x_i - u \qquad \text{for } i \text{ s.t. } x_i - u > 0$$

Challenges

- EVT seems very helpful
  - Characterises limiting distributions very succinctly
- But:
  - Limiting distribution may not actually exist
  - Potential unreliability of extrapolation
  - Ignores time-varying nature of the world
  - How do we define where the ‘tail’ starts?



Source: Nematrian

Tail Weighted Maximum Likelihood Estimation (TWMLE)

- Maximum likelihood estimation (MLE) has nice theoretical properties, e.g. asymptotically efficient and unbiased
- A potentially attractive way of targeting a good fit in the tail is thus to:
  - Re-express the overall likelihood function to relate to ordered observations
  - Differentially weight contributions from individual observations to this (re-expressed) likelihood function, giving greater weight to observations more obviously in the relevant tail
  - Gives same answer as traditional MLE in the special case where all observations are given equal weight
- Maybe also allow for time-varying volatility by including in the problem specification an autocorrelation parameter?

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Joint fat-tailed behaviour

- Crucial to the portfolio construction problem

- Can split the probability distribution into two components:

  a) Marginals (i.e. distributions of each individual risk in isolation); and

  b) Copula (i.e. the remainder, the ‘co-dependency’ between risks)

- However

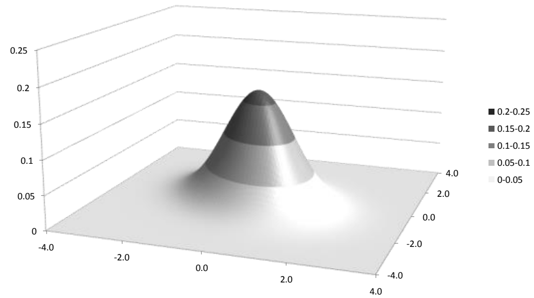
  - Fat-tailed characteristics then difficult to visualise

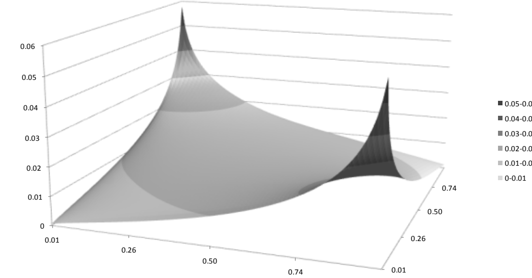
  - Copulas are akin to (indeed are) cumulative distribution functions

  - Many problems depend on a) and b) in tandem

Copulas: a well trodden (mathematical) path

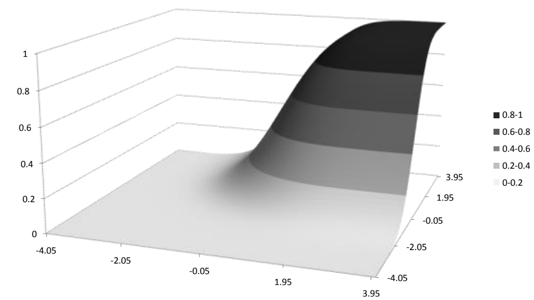
probability density function

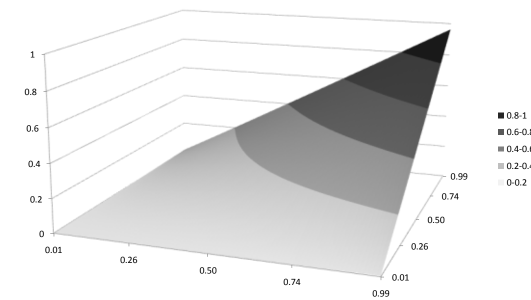




copula density

cumulative distribution function





copula

- The copula involves rescaling (stretching/squashing) each axis so that the distribution is uniform between 0 and 1 along each axis
  - Allows models to exhibit non-zero tail dependency (i.e. ‘correlation’ in tail)

Factor identification

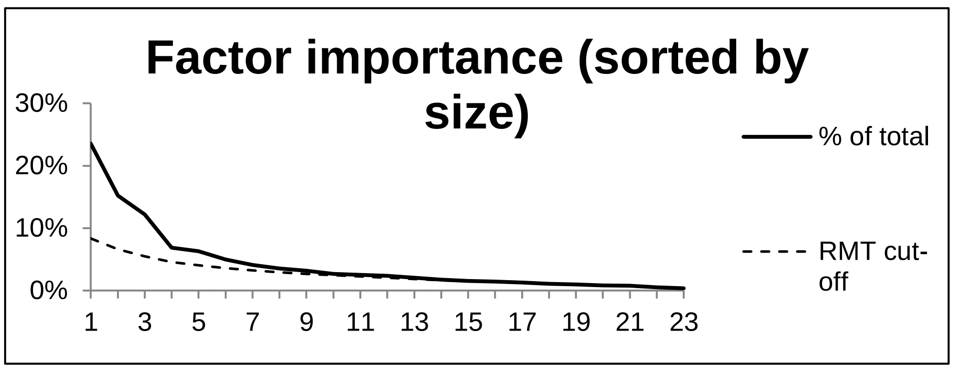
- However, copulas are rather complicated mathematically.
- Typically, simpler correlation based aggregation techniques are used instead (technically involves use of a Gaussian copula)
  - Maybe with adjusted (i.e. higher) correlations if focus is on tail events, to cater for non-zero tail dependencies
- In a portfolio construction context generally involves a factor-based model of the world
  - Vastly reduces number of parameters that need estimating (if large universe)
  - An entire risk model vendor industry focuses on creating and utilising such models

Fundamental data limitations: the 'fine structure' problem

- Suppose we have *N* instruments and estimate the factor structure from *T* observations per instrument where *T* much less than *N* (e.g. as would normally be the case for a whole market model)

- Then at most *T*-1 non-zero factors and random matrix theory (RMT) suggests most of the smaller ones often indistinguishable from ones that would arise randomly

- Places fundamental limits on reliability of factor analysis (or any other risk modelling derived from historic return series, including ones using copulas)



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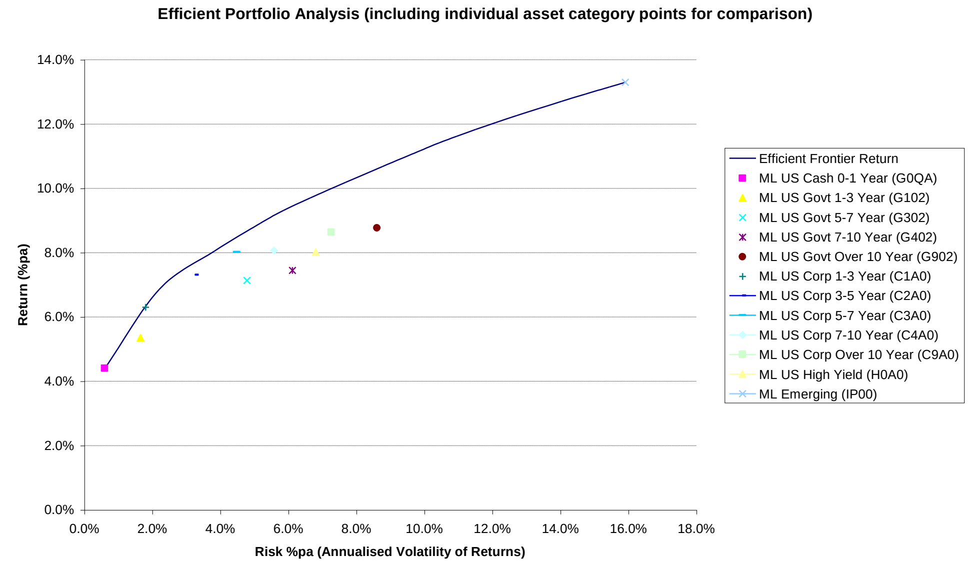
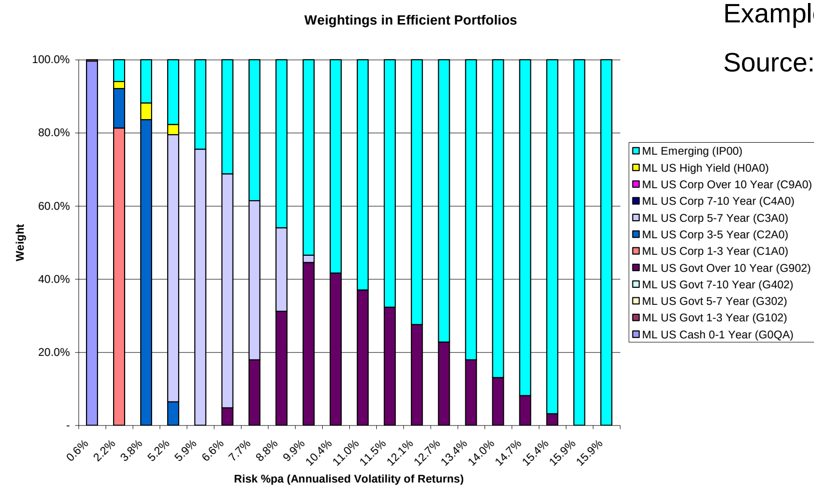
Portfolio construction

- Traditional (quantitative) approach involves portfolio optimisation
- Identify expected return ('alpha') from each position
- Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
- Typically focus on mean-variance optimisation





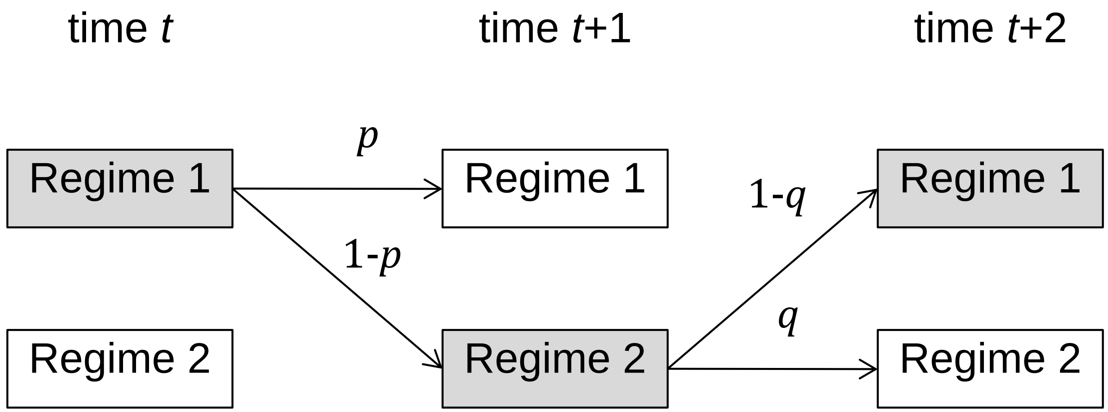
Example for illustration only

Source: Threadneedle

Portfolio construction: sensitivities

- Output results are notoriously sensitive to input assumptions
  - Treat quantitative models with scepticism?
  - Focus on reverse optimisation?
- Techniques proposed to tackle this issue include:
  - Robust approaches and Bayesian priors/anchors, e.g. Black-Litterman
  - Shrinkage
  - Resampled optimisation
- Essentially all suffer from the 'fine structure' problem: the fine structure of optimised portfolio inherently depends on practitioner's (or model creator's) subjective views (or, for e.g. Black-Litterman, how these views are expressed)

Regime switching



- Builds on premise that a high proportion of fat-tailed behaviour observed in practice derives from time-varying nature of the world in which we live
- Usually developed via use of a mathematical technique called Markov chains
- But can develop continuous analogues, e.g. threshold autoregressive models

Impact on asset allocation problem

- Adds a lot of mathematical complexity including dependency on how reliably you can tell which ‘regime’ you are in and what its characteristics are

- Asset allocation problem becomes more utility dependent

  a) The overall conditional distributional form is no longer multivariate Normal;

  b) The investor’s utility function is no longer equivalent, in terms of the portfolio weights derived from it, to a quadratic utility function (means and covariance matrix) as per traditional mean-variance optimisation

- If your utility function is more complex then so will be those of others, making them more sensitive to behavioural factors, and making it more difficult to predict how they might change through time

Avoiding undue complexity?

- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
  - Calculate covariance matrix between return series after stripping out effect of time-varying volatility?
  - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
  - Adjust risk aversion/risk budget appropriately and then unravel time-varying volatility adjustment
  - Or reverse optimise using implied alphas derived from adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed
- Capturing other sources of fat-tailed behaviour adds more complexity

Summary

- *Why are return series often ‘fat tailed’?*
  - Time-varying world in which we live
  - Behavioural dynamics including crowded trades and ‘selection’ effects
- *Extreme Value Theory (EVT) and possible refinements*
  - Extrapolation is inherently challenging, although tail weighted MLE (perhaps adjusted to allow for time varying volatility?) might help
- *Modelling joint fat-tailed behaviour*
  - Implications of the ‘fine structure’ problem
- *Lessons for portfolio construction*
  - Material departure from mean-variance ‘norm’ rapidly increases complexity
  - Problem becomes more sensitive to nature of utility function

Important Information