
Asset Allocation Workshop

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- Taking due account of extreme events is a key issue for (many) market professionals
 - They are a fact of life, can severely disrupt but also provide opportunities for gain
- No amount of clever data analysis can circumvent the need for expert judgement and subjective practitioner input
 - But ignoring mathematical and market insights isn't the right answer
- The limiting case where fat-tailed behaviour is insignificant covers most strands of traditional portfolio construction theory
 - A 2-for-1 opportunity: cover all main strands of basic mean-variance investing, risk budgeting, etc. *as well as* refinements needed to handle extreme events

Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails

■ Chapters:

1. Introduction
2. Fat tails – in single (i.e. univariate) return series
3. Fat tails – in joint (i.e. multivariate) return series
4. Identifying factors that significantly influence markets
5. Traditional portfolio construction techniques
6. Robust mean-variance portfolio construction
7. Regime switching and time-varying risk and return parameters
8. Stress testing
9. Really extreme events

Plus Principles (Chapter 10) and Exercises (Appendix). Each chapter also includes specific sections covering *practitioner perspectives* and *implementation challenges*.

- Book provides practitioners and students with all main recipes (plus author views on them) with a toolkit provided through www.nematrian.com

Selected highlights from “Extreme Events”

1. Introduction

2. Fat tails – in single (i.e. univariate) return series
3. Fat tails – in joint (i.e. multivariate) return series
4. Identifying factors that significantly influence markets

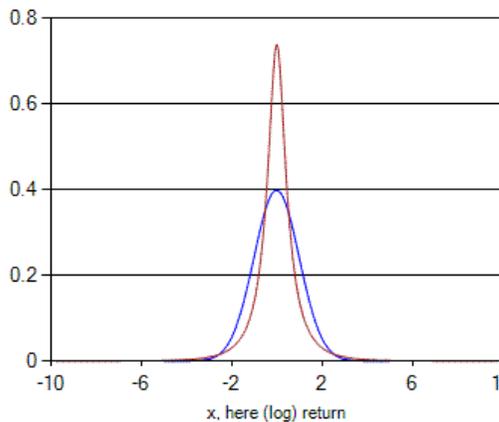
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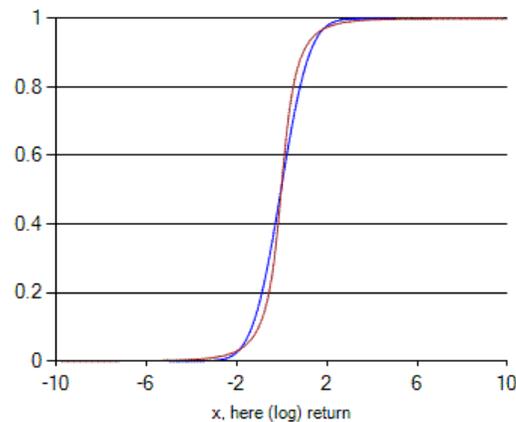
Modelling fat tails for *individual* risks

- ‘Fat-tailed’ means probability of extreme-sized outcomes seems to be higher than if coming from (usually) a (log) Normal distribution
- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below, i.e. QQ-plots
- Note: portfolio construction usually involves *multiple* assets / risk exposures

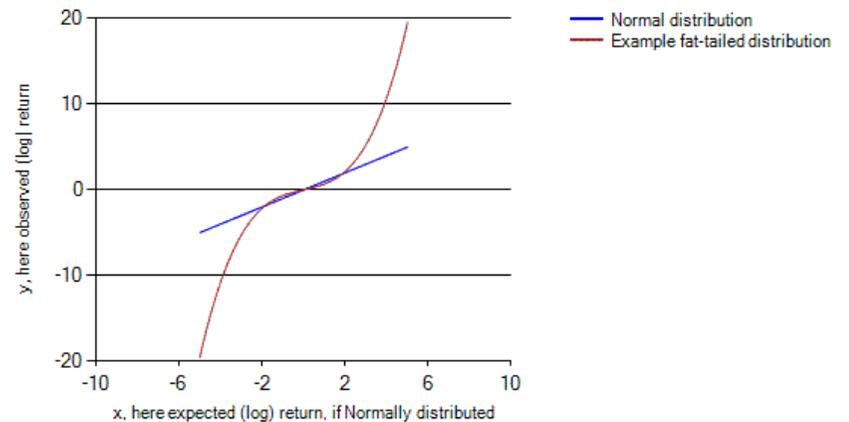
(a) probability density function



(b) cumulative distribution function



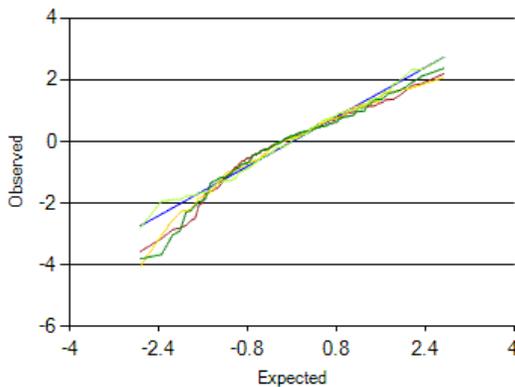
(c) quantile-quantile (QQ) plot



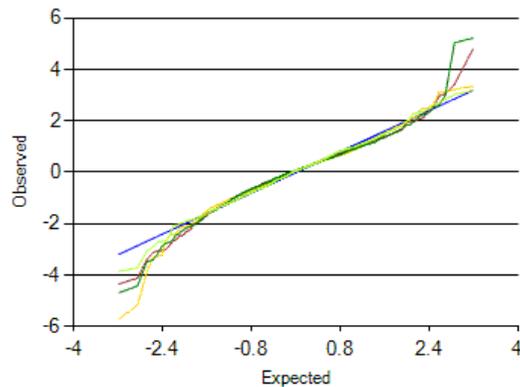
Many (most?) investment return series are 'fat-tailed'

- Some instrument types intrinsically skewed (e.g. high-grade bonds, options)
- Others (e.g. equities) still exhibit fat-tails, particularly higher frequency data

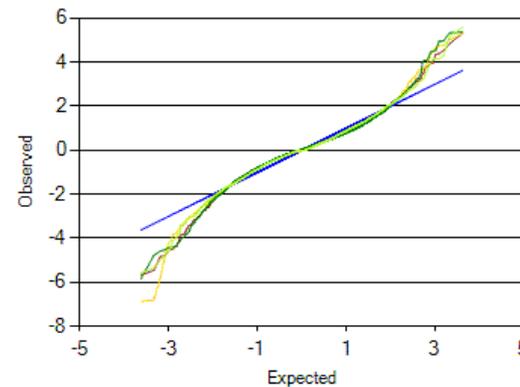
(1) Monthly returns



(2) Weekly returns



(3) Daily returns



— Expected
— FTSE All Share (TRI in GBP)
— S&P 500 Composite (TRI in USD)
— FTSE W Europe Ex UK (TRI in EUR)
— Topix (TRI in JPY)

Source: www.nematrian.com, Threadneedle, S&P, FTSE, Thomson Datastream

Returns from end June 1994 to end Dec 2007, charts show standardised logged returns

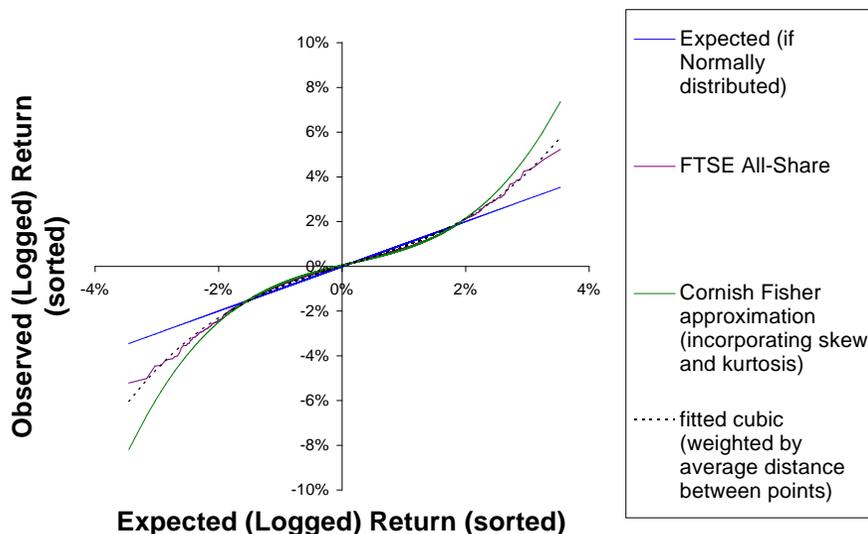
Why are return series often fat-tailed?

- Time-varying nature of the world in which we live
 - Market / sector / instrument volatility (and maybe other distributional characteristics) change through time
- Crowded trades and leverage
- Selection effects, e.g. manager behaviour may (consciously or unconsciously) bias towards fat-tailed behaviour
- As well as intrinsically skewed behaviour such as for individual bonds

Explains some equity index fat tails, particularly upside

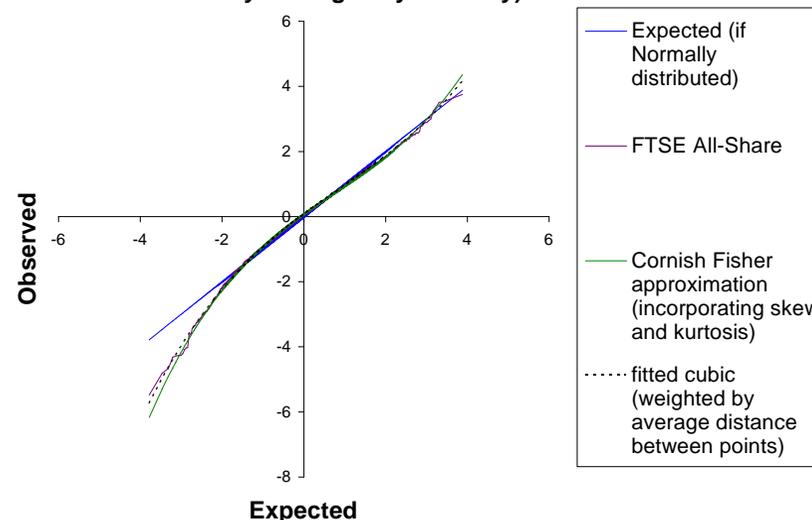
Raw Data

Daily returns (End Jun 1994 to end Dec 2007)



With Short-term Volatility Adjustment

Daily returns (end Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)



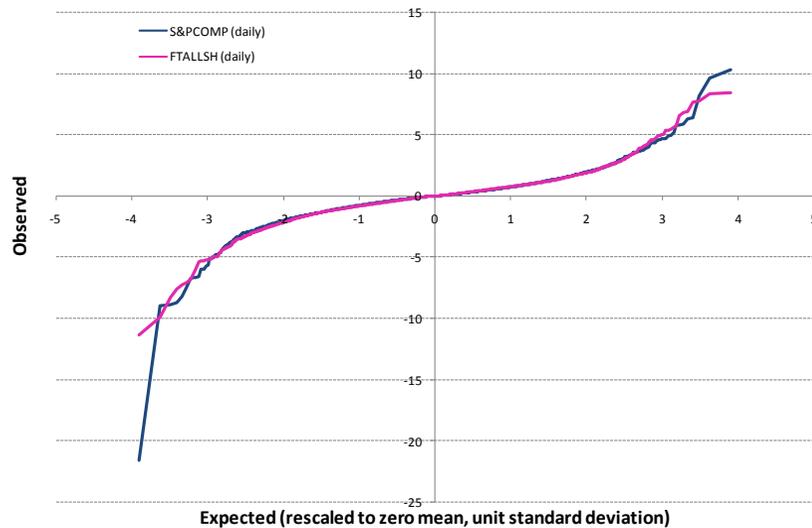
| Average extent to which tail exceeds expected level (average of 6 most extreme outcomes) | | | | |
|--|--------------|-------------|------------|-------------|
| | Downside (%) | | Upside (%) | |
| | Unadj | Adj for vol | Unadj | Adj for vol |
| FTSE All-Share (in GBP) | 54 | 41 | 42 | 3 |
| S&P 500 (in USD) | 68 | 70 | 50 | 7 |
| FTSE Eur ex UK (in EUR) | 48 | 53 | 54 | -3 |
| Topix (in JPY) | 54 | 72 | 42 | 39 |

Source:
Threadneedle, FTSE
Thomson Datastream

And over longer time periods

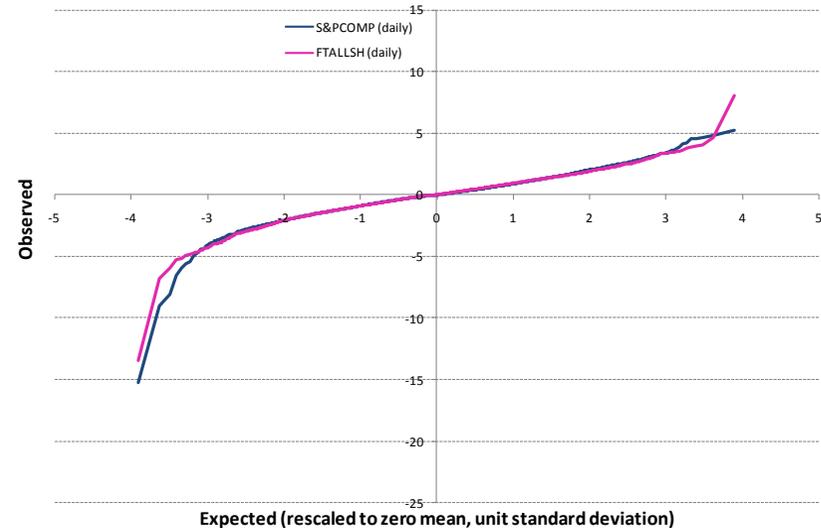
Raw Data

Tail analysis for S&P 500 and FTSE All-Share price movements
31 December 1968 to 24 March 2009



With Short-term Volatility Adjustment

Tail analysis for S&P 500 and FTSE All-Share price movements
(vol adj, by trailing 50 day vol, early 1969 to 24 March 2009)

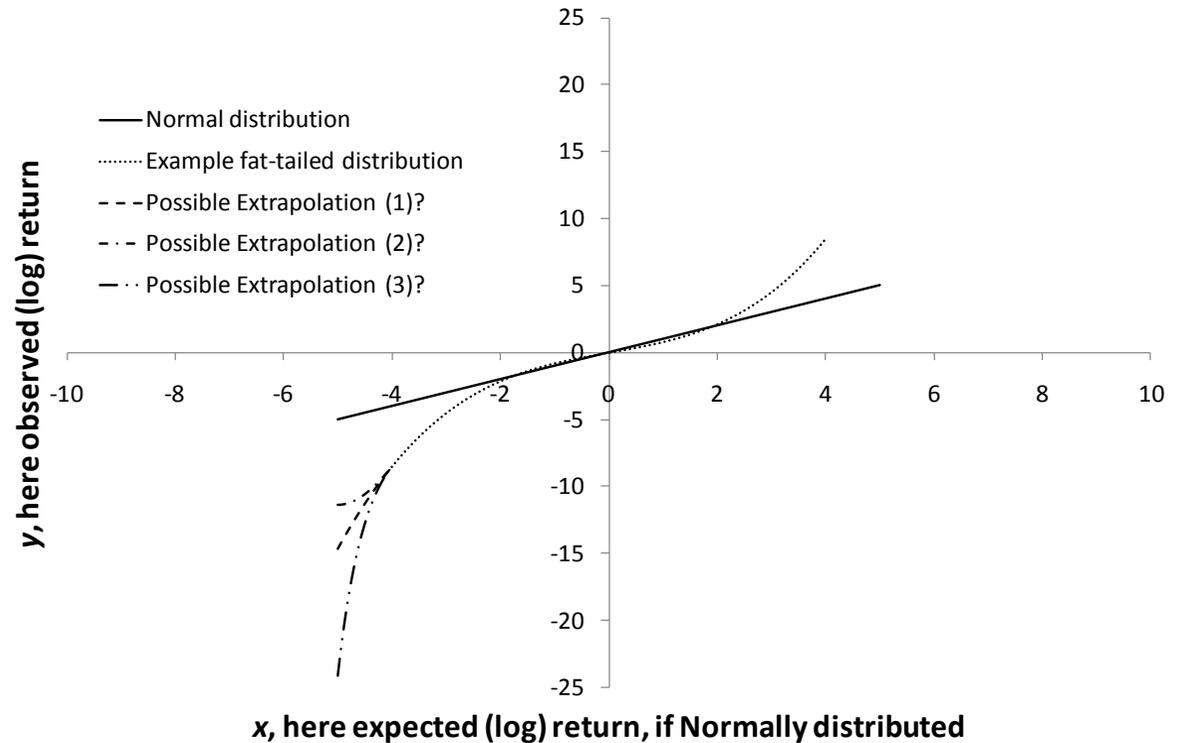


Source: Threadneedle, S&P, FTSE, Thomson Datastream

- Some fat tails still seem to come “out of the blue”
 - E.g. Quant funds in August 2007
 - Too many investors in the same *crowded trades*? Behavioural finance implies potentially unstable
 - For less liquid investments, impact may be via an apparent shift in price basis
- System-wide equivalents via leverage?
 - Leverage introduces/magnifies *liquidity risk*, *forced unwind risk* and *variable borrow cost risk*
 - Like selection, involves behavioural finance effects

- Potentially relevant to risk management (and pricing)
 - Capital adequacy seeks to protect against (we hope) relatively rare events
 - Pricing often dominated by potential magnitude and likelihood of large losses, which are (we hope) rare
 - EVT appears to offer a convenient way of identifying shape of the ‘tail’ distribution, which should be very valuable for such purposes
- But bear in mind
 - Possibility (indeed probability) that the world is not time stationary
 - Inherent unreliability of extrapolation – including extrapolation into the tails of a probability distribution

- EVT seems very helpful
 - Characterises limiting distributions very succinctly
 - But required regularity conditions are potentially strong
 - Relies on existence of a limiting distribution but this is not guaranteed
- At issue is potential unreliability of **extrapolation**



Source: Nematrian

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- 3. Fat tails – in joint (i.e. multivariate) return series**
4. Identifying factors that significantly influence markets

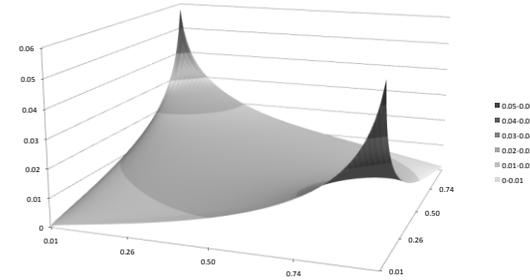
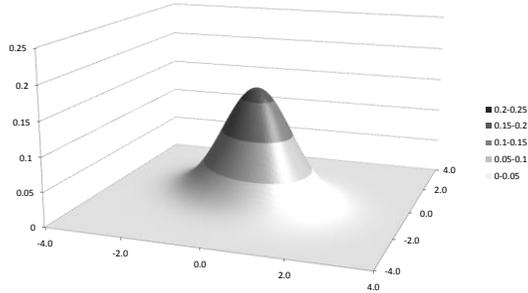
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- Crucial to the portfolio construction problem
- Can split the probability distribution into two components:
 - a) Marginals (i.e. distributions of each individual risk in isolation); and
 - b) Copula (i.e. the remainder, the ‘co-dependency’ between risks)
- However
 - Fat-tailed characteristics then difficult to visualise
 - Copulas are akin to (indeed are) cumulative distribution functions
 - Many problems depend on a) and b) in tandem

Copulas: a well trodden (mathematical) path

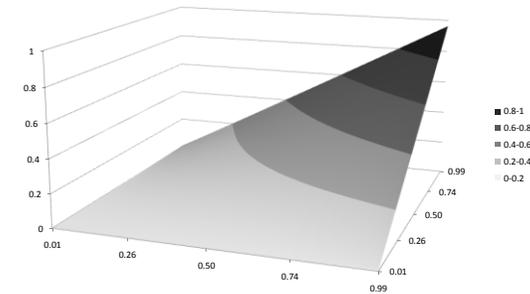
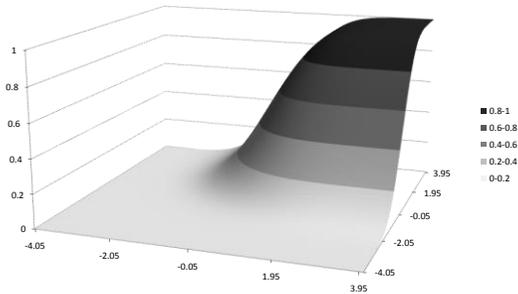
probability density function



copula density



cumulative distribution function



copula

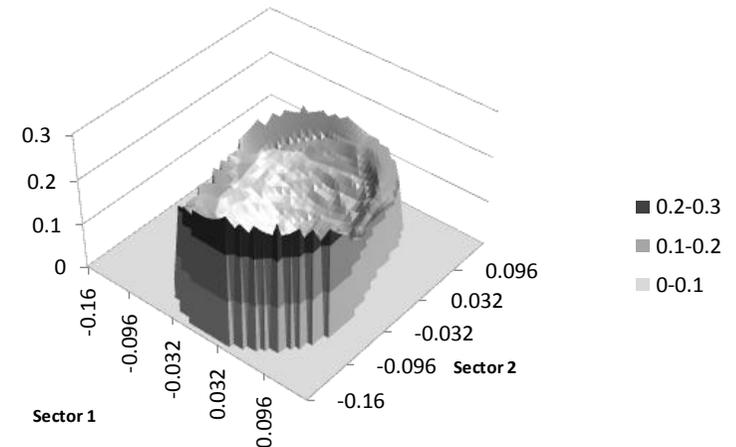
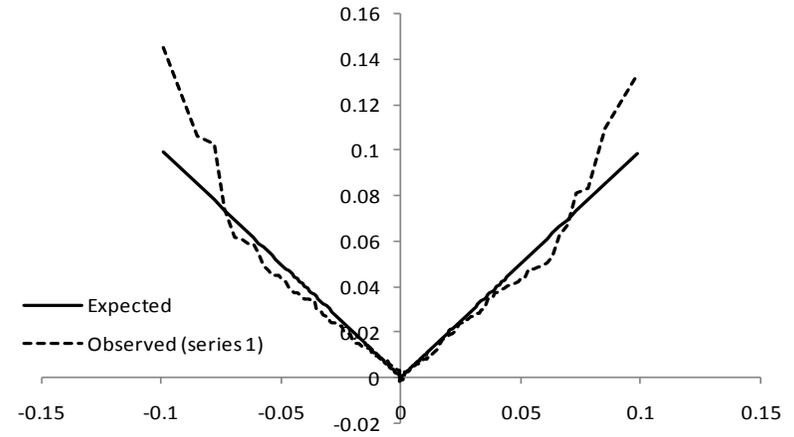


- The copula involves rescaling (stretching/squashing) each axis so that the distribution is uniform between 0 and 1 along each axis
 - Allows models to exhibit non-zero tail dependency (i.e. ‘correlation’ in tail)



But extreme behaviour shows up better in QQ plots

- Book suggests how to refine quantile-quantile plots to show joint (here 2 risk) extreme behaviour
 - Uses 'upwards' QQ plots (right half corresponds to $X_{(i)}$, left half to $-X_{(i)}$)
 - Then create surface plot that encapsulates upwards QQ plots for all (linear) combinations of X and Y, relevant combination given by angle of rotation around centre
 - Encapsulates in a single chart fat-tailed behaviour arising from co-dependency characteristics *and* marginal distributions
 - Like a one-dimensional QQ plot, places greater visual emphasis on extreme events



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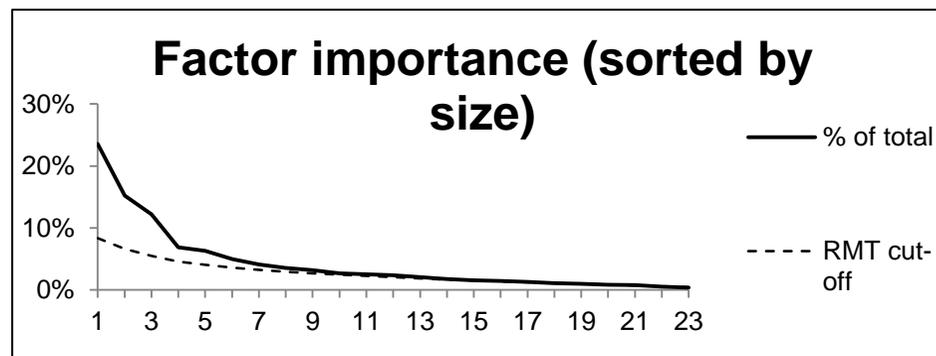
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- Copulas are rather complicated mathematically. Typically simpler correlation based aggregation techniques are used instead
- In a portfolio construction context involves a factor-based model of the world
 - Vastly reduces number of parameters that need estimating (if large universe)
- An entire risk model vendor industry focuses on how to create such models, involving one or more of the following:
 - Fundamental risk models
 - Econometric risk models
 - Statistical risk models

- Suppose we have N instruments and estimate the factor structure from T observations per instrument where T much less than N (e.g. as would normally be the case for a whole market model)
 - Then at most $T-1$ non-zero factors and random matrix theory (RMT) suggests most of the smaller ones often indistinguishable from ones that would arise randomly
 - Places fundamental limits on reliability of factor analysis (or any other risk modelling derived from historic return series)
- Means that fine structure of an optimised portfolio inherently depends on practitioner's (or model creator's) subjective views



- ‘Selection’ effects are common in finance, e.g. annuitants typically have higher than average life expectancies
- Can also apply to portfolios being analysed by risk models
 - Many risk models assume behaviour that is (approximately) Gaussian, i.e. multivariate (log) Normal, akin to lots of different sources of random noise
 - Can decompose multiple series return data into *principal components*, the most important of which contribute the most to the aggregate variability exhibited by securities in the relevant universe
- But what if portfolios are structured to seek ‘meaning’ (e.g. if actively managed!) and ‘meaning’ is (partly) associated with non-Normality?
 - Both *meaning* and *magnitude* are important



- Both are examples of ‘blind source separation’, aiming to identify ‘signals’ (i.e. sources / factors) that explain (observed) market behaviour
- Principal Components Analysis (PCA)
 - Seeks to identify the largest contributors to variance, i.e. **magnitude** of impact
 - ‘Signals’ maximise **sum of variances** of returns of each security within universe
- Independent Components Analysis (ICA)
 - Seeks to identify contributors to market behaviour that are **meaningful**
 - ‘Signals’ maximise **independence**, **non-Normality** and/or **complexity**



Selection effects are potentially very important

| | PCA, only StDev | | | Blend (c = 0.39) | | | ICA, Only Kurtosis | | |
|-------------------|-----------------|------|------------|------------------|------|------------|--------------------|------|------------|
| Component | StDev | Kurt | CF4 est | StDev | Kurt | CF4 est | StDev | Kurt | CF4 est |
| 1st | 10.6% | 3.1 | 23% | 8.3% | 14.9 | 57% | 4.5% | 24.2 | 47% |
| Av (top 6) | 5.9% | 1.6 | 10% | 5.3% | 16.8 | 40% | 4.7% | 18.5 | 40% |

- Assume Cornish-Fisher (CF4) is a good estimator of '1 in 200' risk level
 - Risk then proportional to $std \cdot (1 + c \cdot kurt)$ where $c = 0.39$ at '1 in 200' level
 - *Principal Components Analysis* (PCA) focuses just on standard deviation, *Independent Components Analysis* (ICA) just on kurtosis, blend on both
 - Analysis based on monthly MSCI World sector relative returns Jun 1996 to Feb 2009
- Sizes of '1 in 200' events potentially underestimated several-fold by PCA, if portfolio built on the basis of 'meaning' (e.g. if actively managed)



- Banks that failed during 2007-09 Credit Crisis were disproportionately biased towards strategies that depended on continuing favourable liquidity conditions
- Liquidity risk is highly skewed, i.e. highly fat-tailed
- I.e. these banks were (consciously or unconsciously) biasing their business strategies towards ones that had fat-tailed characteristics
- No wonder traditional risk models appear to have underestimated potential magnitudes of adverse outcomes!

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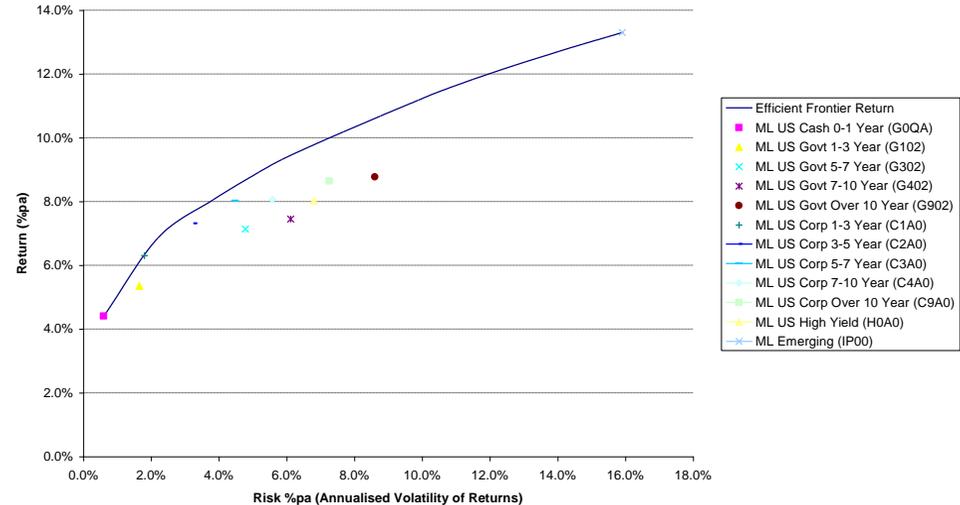
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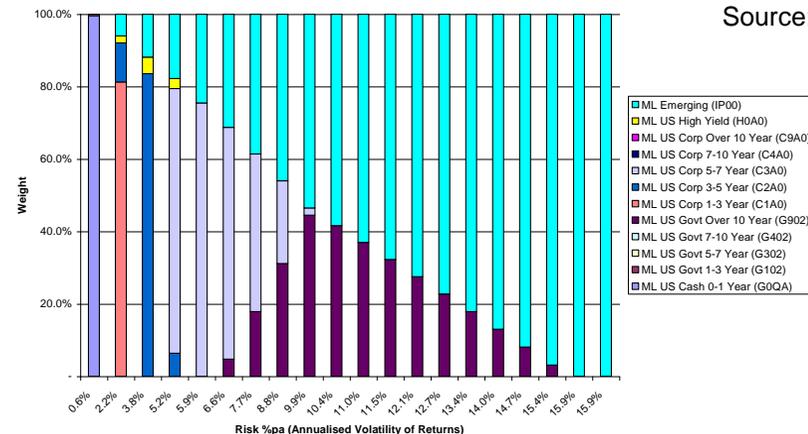
- Traditional (quantitative) approach involves **portfolio optimisation**
 - Identify expected return ('alpha') from each position
 - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
 - Typically focus on mean-variance optimisation

Efficient Portfolio Analysis (including individual asset category points for comparison)



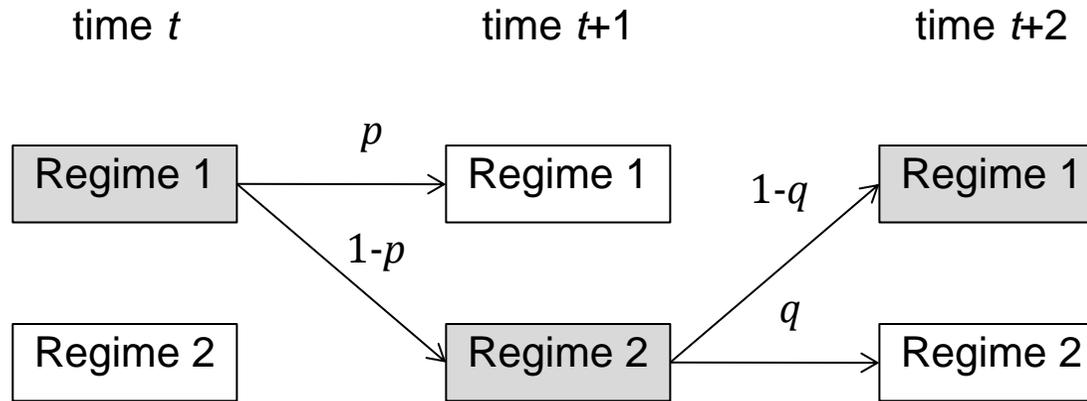
Example for illustration only
Source: Threadneedle

Weightings in Efficient Portfolios



- Output results are notoriously sensitive to input assumptions. Possible responses include:
 - Treat quant models with scepticism (the fundamental manager's approach?)
 - Focus on reverse optimisation
- Book covers all the main quantitative refinements, including:
 - Robust approaches and Bayesian priors/anchors, e.g. Black-Litterman
 - Shrinkage
 - Resampled optimisation
- And ties them back to earlier chapters
 - E.g. how resampled optimisation doesn't avoid 'fine structure' problem, instead it just inherits it from the dataset being used for bootstrapping purposes





- Regime-switching covered in some detail in book
- Builds on premise that a high proportion of fat-tailed behaviour observed in practice derives from time-varying nature of the world in which we live

- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
 - Calculate covariance matrix between return series after stripping out effect of time-varying volatility?
 - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
 - Adjust risk aversion/risk budget appropriately
 - Then unravel time-varying volatility adjustment
 - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed

- Model with a (time-stationary?) mixture of multivariate Normal distributions, see e.g. Scherer (2007)
 - If time-varying then involves regime switching
 - Even more difficult to estimate reliably
- Or use lower partial moments
 - But also challenging to estimate reliably

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- Even a model with good back-test properties may fail to model future risks effectively
 - Because the past is not necessarily a good guide to the future. Markets are not like precisely defined external physical systems whose actions are perfectly predictable.
 - **Magnitude** of impact of a given scenario usually (relatively) easy to calculate
 - The challenge is how to identify the scenario's **likelihood**
- Stress testing focuses **more on magnitude**, and what makes the scenario adverse, and pays **less attention to likelihood**

- There are several types of stress test each with different nuances:
 - Analysis of impact on portfolio (or firm) of movements in specific market drivers, within an envelope of the plausible range of outcomes
 - Specific industry-wide stresses mandated by the regulator (e.g. used in capital computations)
 - A focus on configurations of events that might lead to large losses (e.g. reverse stress testing)
- Some commentators encourage us to revert to a more statistical emphasis when choosing stress tests
- Challenge: **portfolio construction must ultimately trade off risk versus reward, so needs to include some link back to likelihood of occurrence**



- We need to promote the right **mind-set** as there is insufficient data to allow rigorous mathematical analysis
 - Think outside the box (c.f. reverse stress testing)
 - Accept that (Knightian) uncertainty is a fact of life
 - But bear in mind that market behaviour can inform us about how markets react to intrinsic uncertainty and that there is typically a premium for flexibility
 - Be particularly aware of exposures that are sensitive to aggregate ‘market risk appetite’, including liquidity risk and other selection-sensitive risk types
 - Don’t undervalue good governance and operational management (including ERM principles)
 - Remember markets are driven in part by (hard to quantify) behavioural factors



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