
Applying EVT and alternatives to portfolio construction and the management of risk

**Presentation to Institute and Faculty of Actuaries
Open Forum on Extreme Value Theory**

Malcolm Kemp

26 January 2012

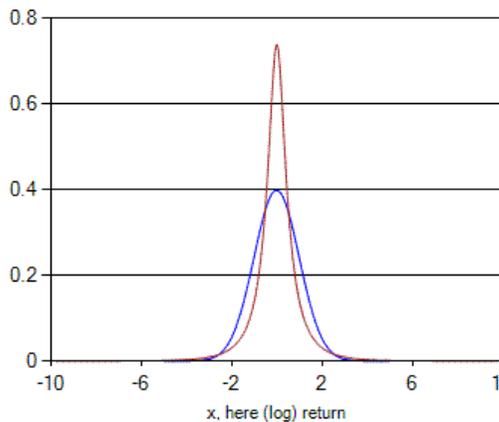
- Why are return series often ‘fat tailed’?
- Strengths and weaknesses of Extreme Value Theory (EVT)
- Interaction with portfolio construction

- See also:
 - Kemp, M.H.D. (2010). *Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails*. John Wiley & Sons
 - Toolkit, charts etc. on www.nematrian.com/extremeevents.aspx

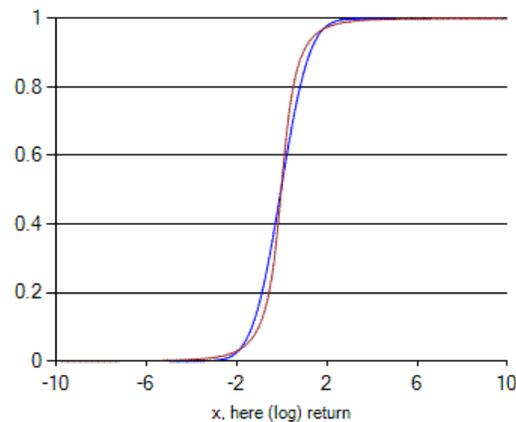
Modelling fat-tailed behaviour for *individual* risks

- ‘Fat-tailed’ means probability of extreme-sized outcomes seems to be higher than if coming from (usually) a (log) Normal distribution
- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below, i.e. QQ-plots
- Note: portfolio construction usually involves *multiple* assets / risk exposures

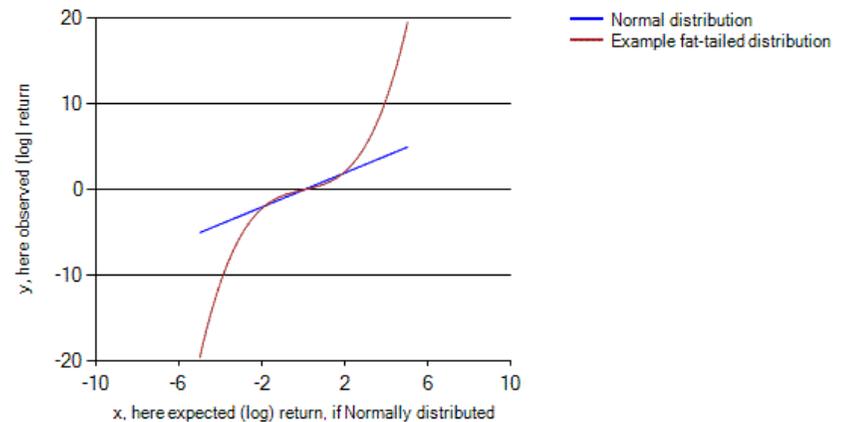
(a) probability density function



(b) cumulative distribution function



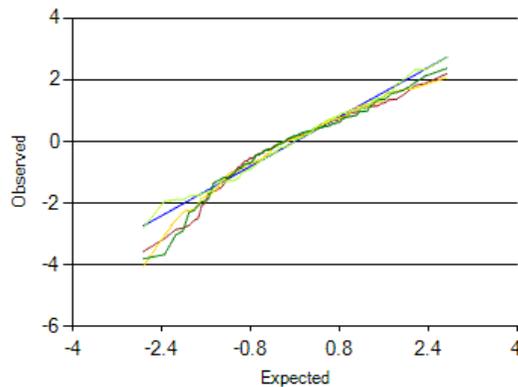
(c) quantile-quantile (QQ) plot



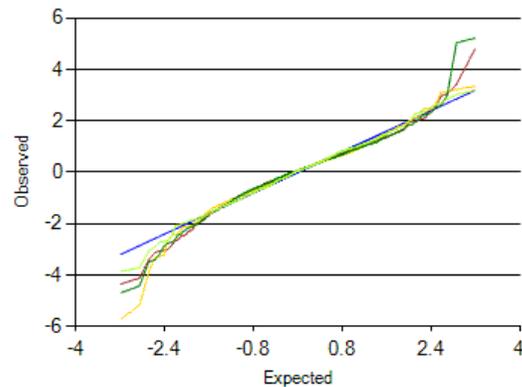
Many (most?) investment return series are 'fat-tailed'

- Some instrument types intrinsically skewed (e.g. high-grade bonds, options)
- Others (e.g. equities) still exhibit fat-tails, particularly higher frequency data

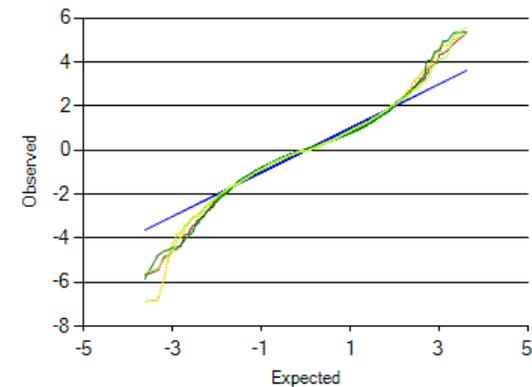
(1) Monthly returns



(2) Weekly returns



(3) Daily returns



— Expected
— FTSE All Share (TRI in GBP)
— S&P 500 Composite (TRI in USD)
— FTSE W Europe Ex UK (TRI in EUR)
— Topix (TRI in JPY)

Source: www.nematrian.com, Threadneedle, S&P, FTSE, Thomson Datastream

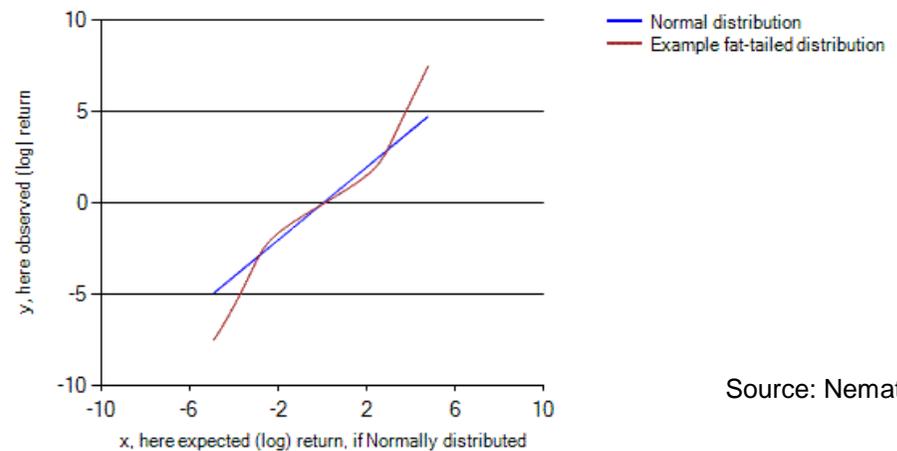
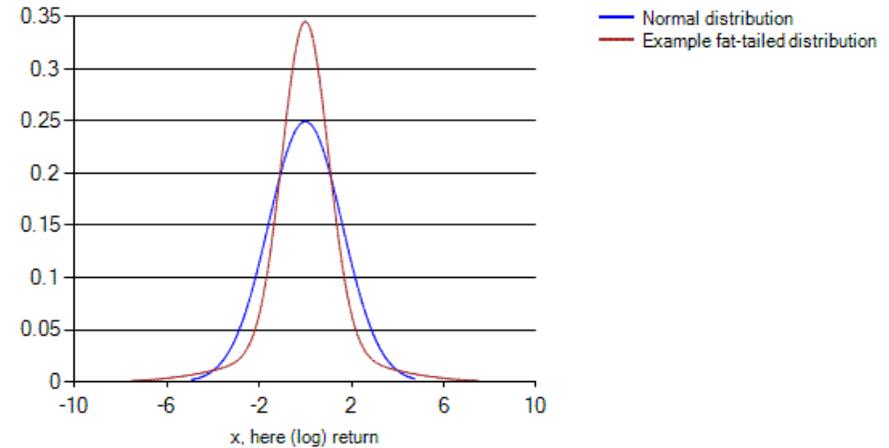
Returns from end June 1994 to end Dec 2007, charts show standardised logged returns

Why are return series often fat-tailed?

- Time-varying nature of the world in which we live
 - Market / sector / instrument volatility (and maybe other distributional characteristics) change through time
 - Heteroscedasticity, GARCH, regime switching
 - Returns may be (*conditionally*) Normal over short time periods, but data series still (*unconditionally*) non-Normal when viewed over longer time periods
- Selection effects, e.g. manager behaviour may (consciously or unconsciously) bias towards fat-tailed behaviour, see Kemp (2010)
- Crowded trades and leverage
- As well as intrinsically skewed behaviour such as for individual bonds

Distributional mixtures of Normal distributions

- E.g. draw X with prob p from N_1 and prob $(1-p)$ from N_2
 - Quite different behaviour to *linear combination mixtures*, i.e. $a.X_1 + b.X_2$
- If N_1 and N_2 have **same** mean but **different** s.d.'s then distributional mixture fat-tailed (if $p \neq 0$ or 1) but linear combination mixture isn't.
- Time-varying volatility is similar, involves draws from different distributions at different times

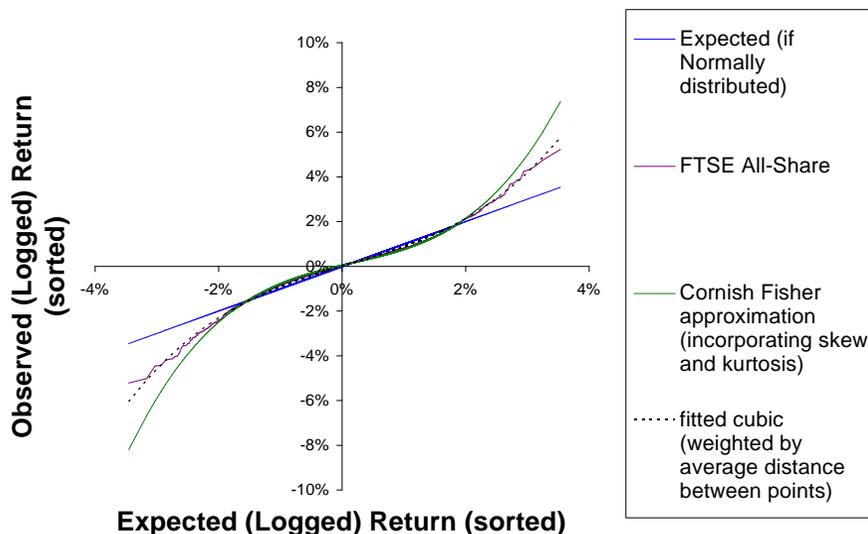


Source: Nematrian

Explains some equity index fat tails, particularly upside

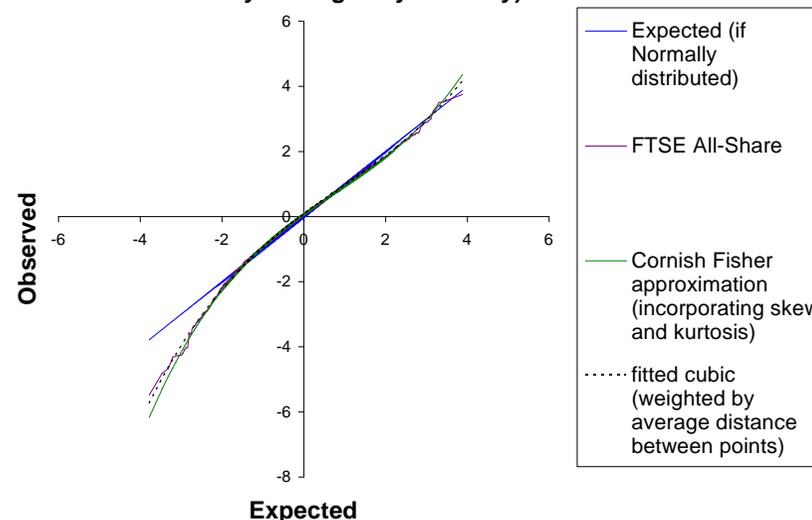
Raw Data

Daily returns (End Jun 1994 to end Dec 2007)



With Short-term Volatility Adjustment

Daily returns (end Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)



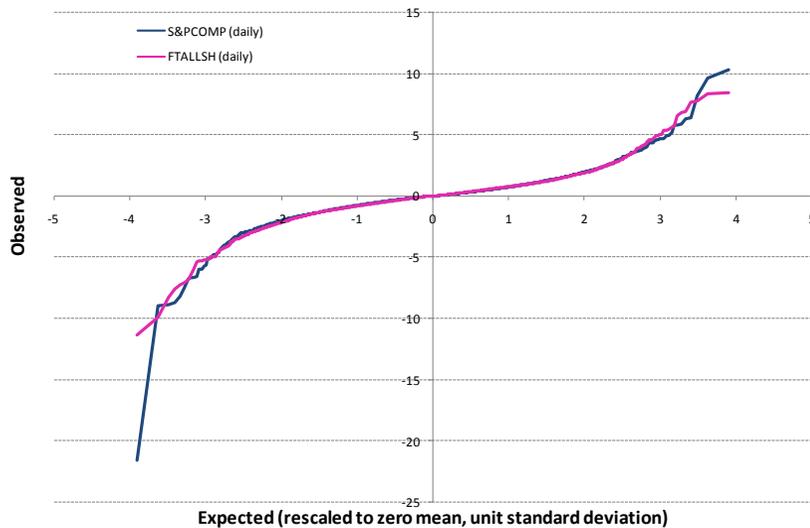
Average extent to which tail exceeds expected level (average of 6 most extreme outcomes)				
	Downside (%)		Upside (%)	
	Unadj	Adj for vol	Unadj	Adj for vol
FTSE All-Share (in GBP)	54	41	42	3
S&P 500 (in USD)	68	70	50	7
FTSE Eur ex UK (in EUR)	48	53	54	-3
Topix (in JPY)	54	72	42	39

Source:
Threadneedle, FTSE
Thomson Datastream

And over longer time periods

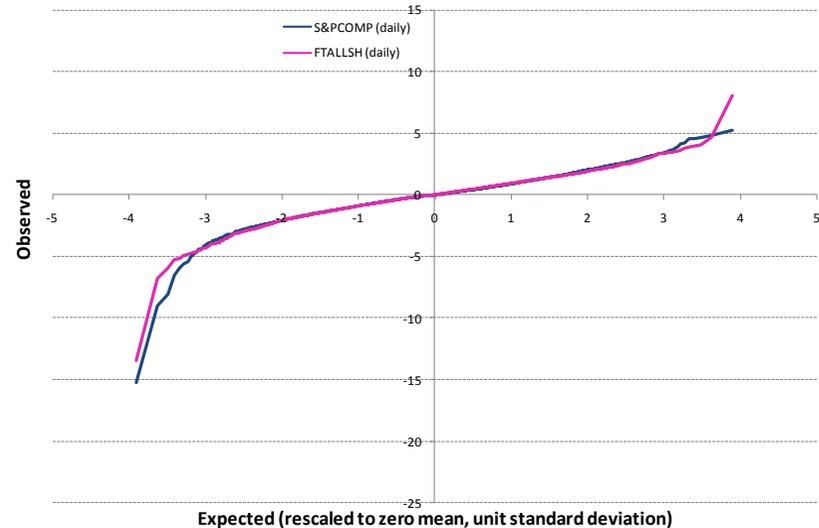
Raw Data

Tail analysis for S&P 500 and FTSE All-Share price movements
31 December 1968 to 24 March 2009



With Short-term Volatility Adjustment

Tail analysis for S&P 500 and FTSE All-Share price movements
(vol adj, by trailing 50 day vol, early 1969 to 24 March 2009)



Source: Threadneedle, S&P, FTSE, Thomson Datastream

- Some fat tails still seem to come “out of the blue”
 - E.g. Quant funds in August 2007
 - Too many investors in the same *crowded trades*? Behavioural finance implies potentially unstable
 - For less liquid investments, impact may be via an apparent shift in price basis
- System-wide equivalents via leverage?
 - Leverage introduces/magnifies *liquidity risk*, *forced unwind risk* and *variable borrow cost risk*
 - Like selection, involves behavioural finance effects

- EVT an enticing prospect
 - Appears to offer a mathematically sound way of identifying shape of the 'tail' of a distribution, and hence identifying likelihood of extreme (i.e. rare) events
 - Capital adequacy seeks to protect against (we hope) relatively rare events
 - Insurance and credit risk pricing can be dominated by potential magnitude and likelihood of large losses, which are also (we hope) rare
- But bear in mind
 - Inherent unreliability of extrapolation – including extrapolation into the tails of a probability distribution
 - Possibility (indeed probability) that the world is not time stationary
 - We may need to consider a multivariate analogue for portfolio construction

- Suppose interested in risk measures relating to losses, x_j
- EVT aims to supply two closely related results:
 1. Distributions of ‘block maxima’ (or ‘block minima’), i.e. maximum value of x_j in blocks of m observations of x (more traditional use of EVT, wasteful of data):

$$m_j \equiv \max \{ x_k : (j-1)m + 1 \leq k \leq jm \}$$

2. Distributions of ‘threshold exceedances’ (i.e. ‘peaks-over-thresholds’), where u is a predetermined high threshold and we focus on realisations of x_j that exceed u (more relevant to, e.g. computation of Value at Risk i.e. VaR), i.e.:

$$y_i \equiv x_i - u \quad \text{for } i \text{ s.t. } x_i - u > 0$$

Main result for block maxima

- Suppose that $x_1, x_2, \dots, x_n, \dots$ are independent random variables possessing same cumulative distribution function, F , and that there exist sequences a_n and b_n such that the following tends in distributional form to Q , a non-degenerate probability distribution from which random variable y is drawn

$$\frac{\max \{x_1, \dots, x_n\} - b_n}{a_n} \xrightarrow{D} y$$

- Then Q is equal to $H(\xi)$ for some ξ (if a_n and b_n appropriately scaled) where $H(\xi)$ is the generalised extreme value (GEV) distribution. F is then said to be in the maximum domain of attraction of $H(\xi)$

value of $\xi = 1/\alpha$	GEV sub-type	(cumulative) distribution function
$\xi = 0$	Gumbel	$\exp(-\exp(-x))$ for $-\infty < x < \infty$
$\xi > 0$	Fréchet	$\exp(-(1 + \xi x)^{-1/\xi})$ for $1 + \xi x > 0$, otherwise 0
$\xi < 0$	Weibull	$\exp(-(1 - \xi x)^{1/\xi})$ for $1 - \xi x > 0$, otherwise 1



■ Let F_u be defined as follows: $F_u(z) \equiv \Pr(x - u < z | x > u)$

■ Then under same hypotheses as applied to block maxima we have:

$$\lim_{u \rightarrow x_f} \sup_{0 < z < x_f - u} |F_u(z) - G_{\mu, \sigma, \xi}(z)| = 0$$

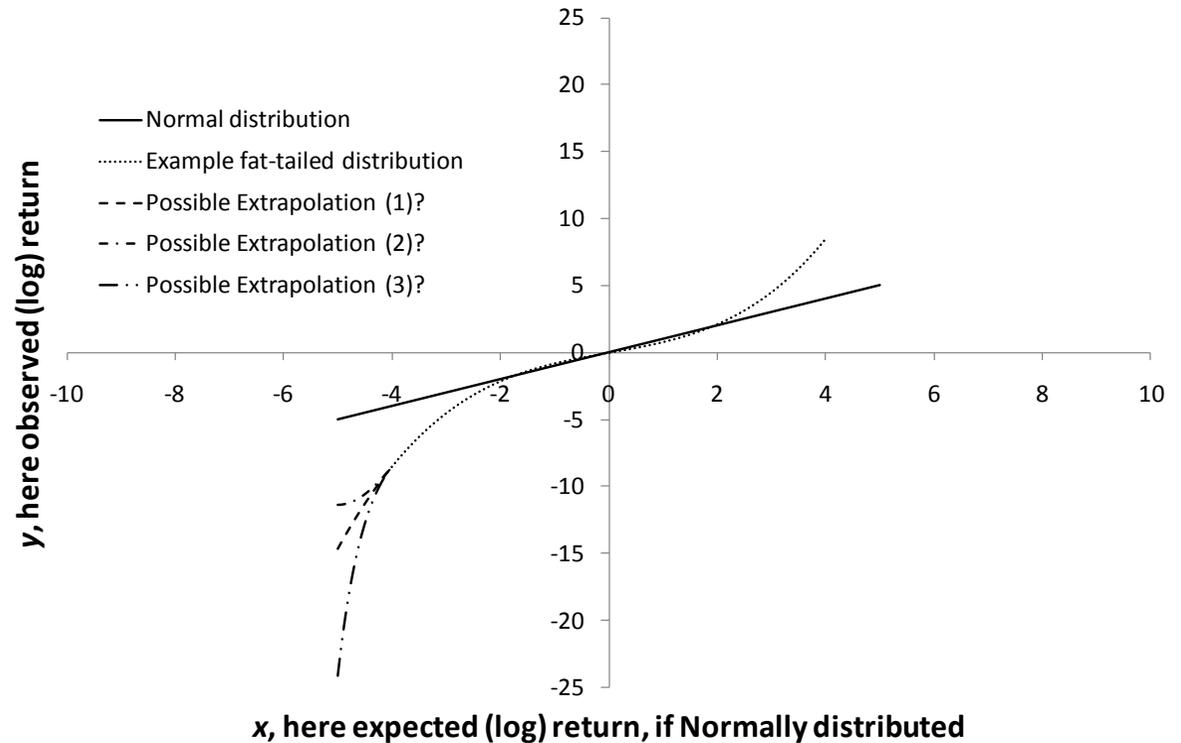
■ Where $G_{\mu, \sigma, \xi}(z)$ has the form:

$$G_{\mu, \sigma, \xi}(z) = \begin{cases} 1 - \left(1 + \xi \frac{(z - \mu)}{\sigma}\right)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{(z - \mu)}{\sigma}\right), & \xi = 0 \end{cases}$$

■ Here ξ has the same type of meaning as before, e.g. $G_{\mu, \sigma, \xi}(z)$ is in the maximum domain of attraction of $H(\xi)$



- EVT seems very helpful
 - Characterises limiting distributions very succinctly
 - But requires regularity conditions that may not be satisfied
 - Relies on existence of a limiting distribution but this is not guaranteed
- At issue is potential unreliability of **extrapolation**
 - E.g. Press et al. (2007)



Source: Nematrian

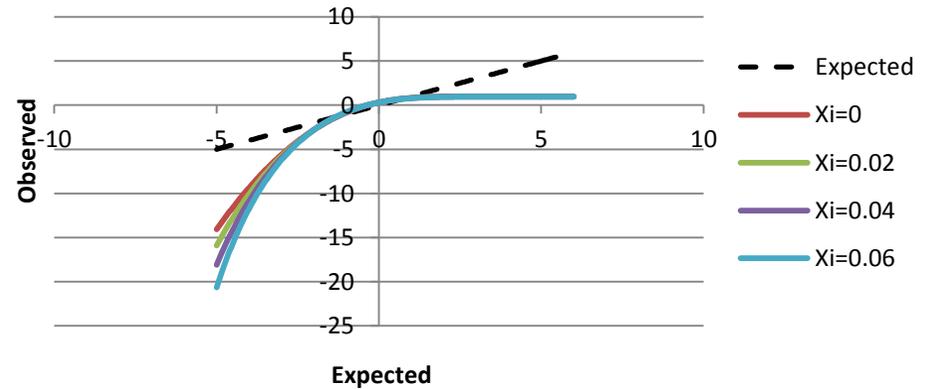
- Assume limiting distribution of tail is fat-tailed GPD
 - Thus use approximation: $F_u(y) \approx G_{\mu, \sigma, \xi}(y)$
 - Problem of estimating F and its (tail) quantiles then reduces to problem of estimating μ , σ and ξ for the approximating generalised Pareto distribution
 - Can be done using mean excess functions, maximum likelihood (ML) estimation, method of moments etc.
- But equally we could fit to the relevant part of the QQ-plot using any other reasonable form of curve fitting approach
 - E.g. polynomial curve fit such as a cubic (see earlier), as long as the resulting extrapolation is credible



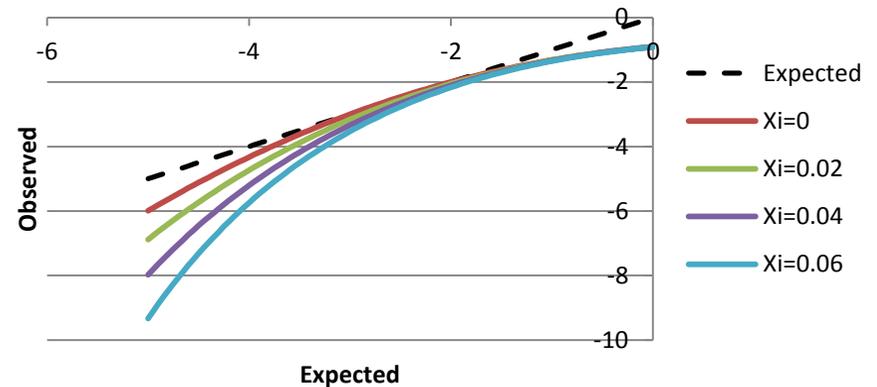
Some subtleties of EVT

- QQ-plot of GPD is convex upwards (if $\xi \geq 0$)
- If data is Normal then fitting a GPD may overstate size of extreme events somewhat
 - Since Normal has same tail characteristics as $\xi = 0$ GPD
- In real life often have multiple series / loss types
 - Can construct *multivariate* EVT theory, but more complex
 - E.g. McNeil, Frey & Embrechts (2005)

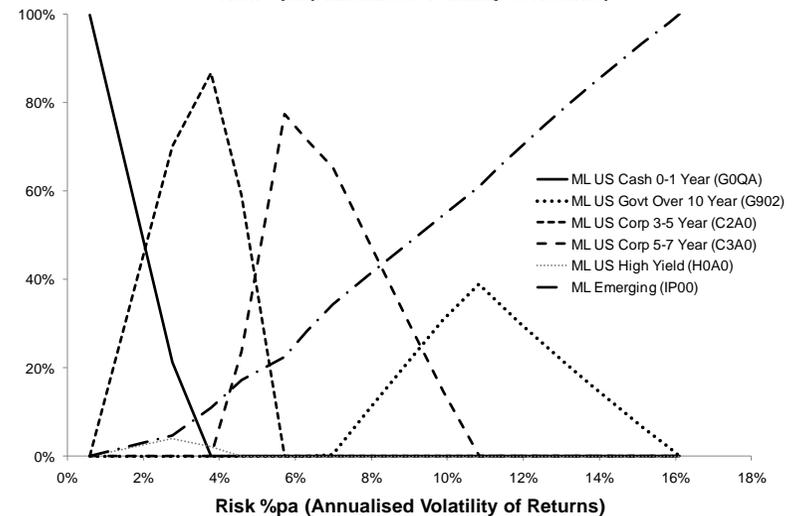
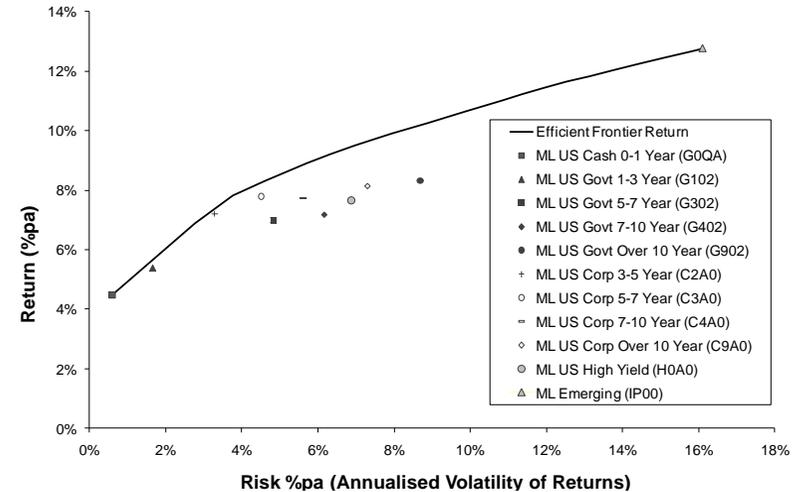
QQ-plot, entire GPD (versus Normal)



QQ-plot versus N(x) fitted to -2 and -3



- Traditional (quantitative) approach involves **portfolio optimisation**
 - Typically mean-variance optimisation
 - Identify expected return ('alpha') from each position
 - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
 - Maximise $a.r - \lambda.a^T V a$
- Intrinsically **multivariate**



Source: Nematrian

- Output results are notoriously sensitive to input assumptions
- Possible responses:
 - Treat quant models with scepticism (the fundamental manager's approach?)
 - Use robust approaches, Bayesian priors/anchors, e.g.
 - Black-Litterman
 - 'Shrinkage'
 - Position limit 'priors' (e.g. $1/N$, long-only etc.)
 - Resampling
 - Focus on reverse optimisation

- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
 - Calculate covariance matrix between return series after stripping out effect of time-varying volatility
 - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
 - Adjust risk aversion/risk budget appropriately
 - Then unravel time-varying volatility adjustment
 - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed



- Model with a mixture of multivariate Normal distributions (or GPDs, ...)
 - Time-stationary? Maybe not realistic?
 - Time-varying?
 - (Discrete) regime switching, and/or
 - (Continuous) parameterisation (and continuous time?)
- However:
 - Even a mixture of just two multivariate Normal distributions involves estimation of twice as many parameters
 - Results even more sensitive to input assumptions
 - Time varying => dynamic => sensitivity to transaction costs



- Fat-tailed behaviour
 - Very common in practice
 - Several intrinsic reasons for its existence, including time-varying world
- Extreme Value Theory (EVT)
 - Enticingly simple (at least in concept)
 - But subject to same underlying issues as any other form of extrapolation
- Portfolio construction can be refined to cater better for extreme events
 - Adjust for (global) time-varying volatility
 - Any further refinements become very complex



Important Information

Material copyright (c) [Nematrian](#), 2012 unless otherwise stated.

All contents of this presentation are based on the opinions of the relevant Nematrian employee or agent and should not be relied upon to represent factually accurate statements without further verification by third parties. Any opinions expressed are made as at the date of publication but are subject to change without notice.

Any investment material contained in this presentation is for Investment Professionals use only, not to be relied upon by private investors. Past performance is not a guide to future returns. The value of investments is not guaranteed and may fall as well as rise, and may be affected by exchange rate fluctuations. Performance figures relating to a fund or representative account may differ from that of other separately managed accounts due to differences such as cash flows, charges, applicable taxes and differences in investment strategy and restrictions. Investment research and analysis included in this document has been produced by Nematrian for its own purposes and any investment ideas or opinions it contains may have been acted upon prior to publication and is made available here incidentally. The mention of any fund (or investment) does not constitute an offer or invitation to subscribe to shares in that fund (or to increase or reduce exposure to that investment). References to target or expected returns are not guaranteed in any way and may be affected by client constraints as well as external factors and management.

The information contained in this document is confidential and copyrighted and should not be disclosed to third parties. It is provided on the basis that the recipient will maintain its confidence, unless it is required to disclose it by applicable law or regulations. Certain information contained in this document may amount to a trade secret, and could, if disclosed, prejudice the commercial interests of Nematrian or its employees or agents. If you intend to disclose any of the information contained in this document for any reason, including, but not limited to, in response to a request under the Freedom of Information Act or similar legislation, you agree to notify and consult with Nematrian prior to making any such disclosure, so that Nematrian can ensure that its rights and the rights of its employees or agents are protected. Any entity or person with access to this information shall be subject to this confidentiality statement.

Information obtained from external sources is believed to be reliable but its accuracy or completeness cannot be guaranteed.

Any Nematrian software referred to in this presentation is copyrighted and confidential and is provided “as is”, with all faults and without any warranty of any kind, and Nematrian hereby disclaims all warranties with respect to such software, either express, implied or statutory, including, but not limited to, the implied warranties and/or conditions of merchantability, of satisfactory quality, or fitness for a particular purpose, of accuracy, of quiet enjoyment, and non-infringement of third party rights. Nematrian does not warrant against interference with your enjoyment of the software, that the functions contained in the software will meet your requirements, that the operation of the software will be uninterrupted or error-free, or that defects in the software will be corrected. For fuller details, see license terms on www.nematrian.com. Title to the software and all associated intellectual property rights is retained by Nematrian and/or its licensors.