# Applying EVT and alternatives to portfolio construction and the management of risk

**Presentation to Institute and Faculty of Actuaries Open Forum on Extreme Value Theory**

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# Using EVT and alternatives



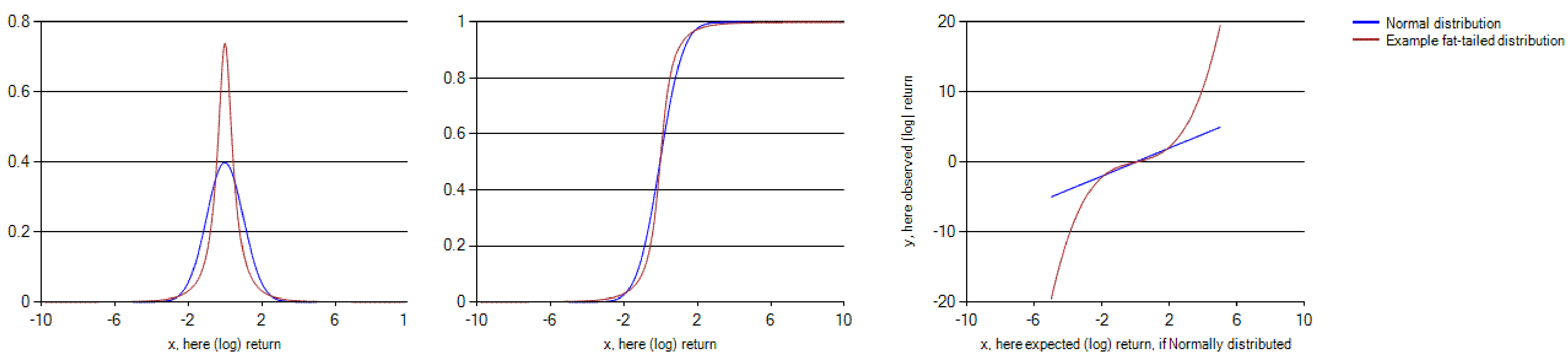
- Why are return series often 'fat tailed'?
- Strengths and weaknesses of Extreme Value Theory (EVT)
- Interaction with portfolio construction
- See also:
  - Kemp, M.H.D. (2010). *Extreme Events: Robust Portfolio Construction in the Presence of Fat Tails*. John Wiley & Sons
  - Toolkit, charts etc. on www.nematrian.com/extremeevents.aspx

# Modelling fat-tailed behaviour for *individual* risks



- ‘Fat-tailed’ means probability of extreme-sized outcomes seems to be higher than if coming from (usually) a (log) Normal distribution
- There are various ways of visualising fat tails in a *single* return distribution. Easiest to see in format (c) below, i.e. QQ-plots
- Note: portfolio construction usually involves *multiple* assets / risk exposures

(a) probability density function | (b) cumulative distribution function | (c) quantile-quantile (QQ) plot




Nematrian

Source: [www.nematrian.com](http://www.nematrian.com)

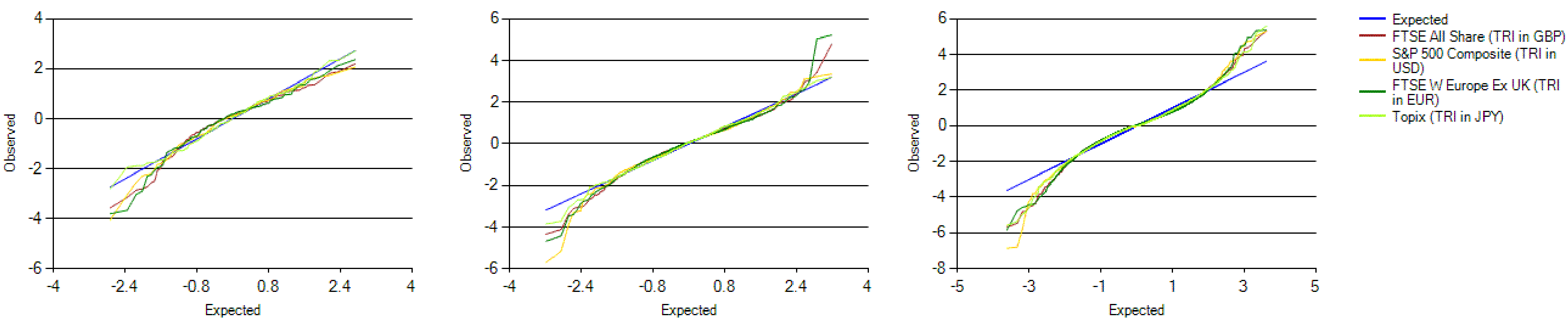
# Many (most?) investment return series are ‘fat-tailed’

- Some instrument types intrinsically skewed (e.g. high-grade bonds, options)
- Others (e.g. equities) still exhibit fat-tails, particularly higher frequency data

(1) Monthly returns

(2) Weekly returns

(3) Daily returns




Source: [www.nematrian.com](http://www.nematrian.com), Threadneedle, S&P, FTSE, Thomson Datastream

Returns from end June 1994 to end Dec 2007, charts show standardised logged returns

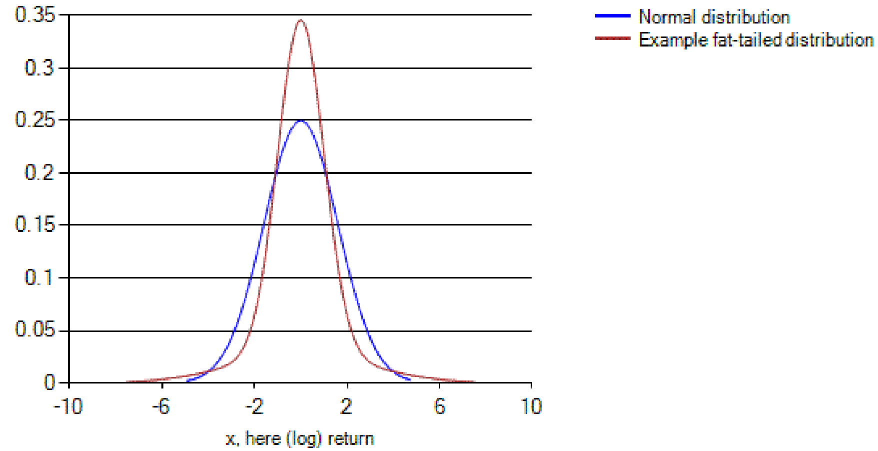
# Why are return series often fat-tailed?

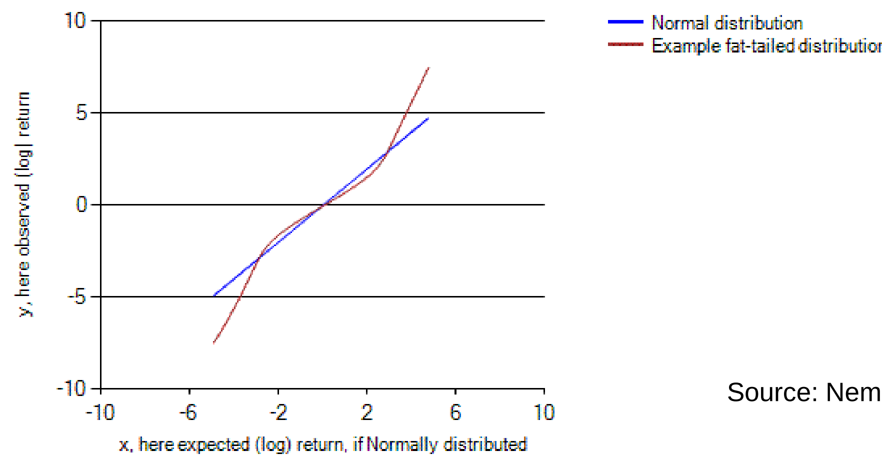


- Time-varying nature of the world in which we live
  - Market / sector / instrument volatility (and maybe other distributional characteristics) change through time
    - Heteroscedasticity, GARCH, regime switching
  - Returns may be (*conditionally*) Normal over short time periods, but data series still (*unconditionally*) non-Normal when viewed over longer time periods
- Selection effects, e.g. manager behaviour may (consciously or unconsciously) bias towards fat-tailed behaviour, see Kemp (2010)
- Crowded trades and leverage
- As well as intrinsically skewed behaviour such as for individual bonds

# Distributional mixtures of Normal distributions

- E.g. draw $X$ with prob $p$ from $N_1$ and prob (1-$p$) from $N_2$
  - Quite different behaviour to *linear combination mixtures*, i.e. $a.X_1 + b.X_2$
- If $N_1$ and $N_2$ have same mean but different s.d.'s then distributional mixture fat-tailed (if $p \neq 0$ or 1) but linear combination mixture isn't.
- Time-varying volatility is similar, involves draws from different distributions at different times



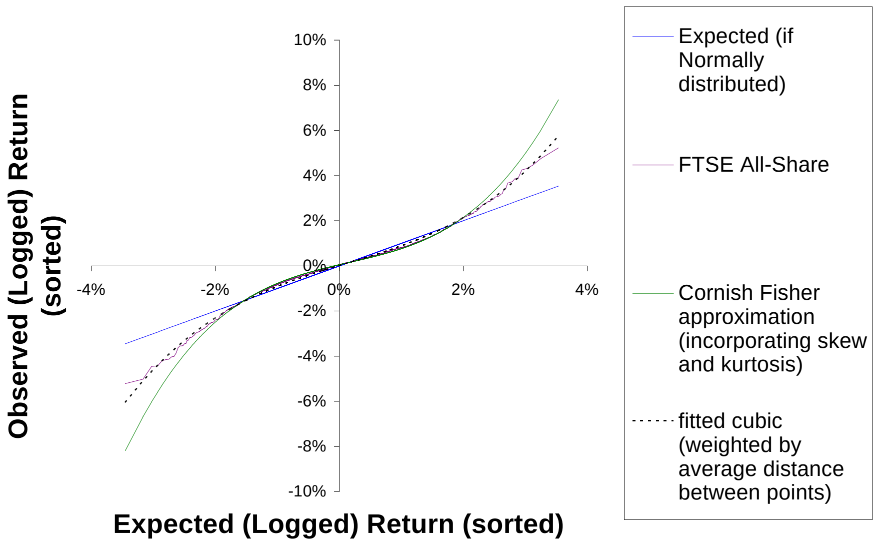

Source: Nematrian

# Explains some equity index fat fails, particularly upside



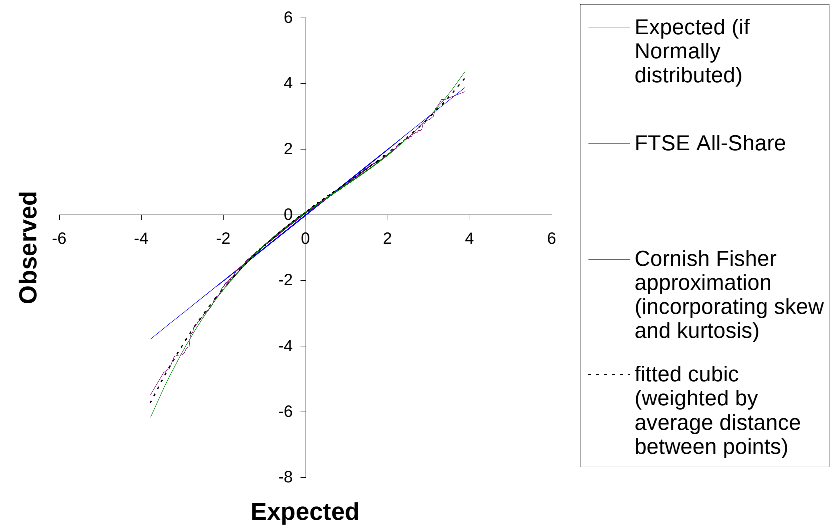
## Raw Data

**Daily returns (End Jun 1994 to end Dec 2007)**




## With Short-term Volatility Adjustment

**Daily returns (end Jun 1994 to end Dec 2007, scaled by 50 business day trailing daily volatility)**




| Average extent to which tail exceeds expected level (average of 6 most extreme outcomes) | | | | |
|---|---|---|---|---|
| | Downside (%) | | Upside (%) | |
| | Unadj | Adj for vol | Unadj | Adj for vol |
| FTSE All-Share (in GBP) | 54 | 41 | 42 | 3 |
| S&P 500 (in USD) | 68 | 70 | 50 | 7 |
| FTSE Eur ex UK (in EUR) | 48 | 53 | 54 | -3 |
| Topix (in JPY) | 54 | 72 | 42 | 39 |

Source:

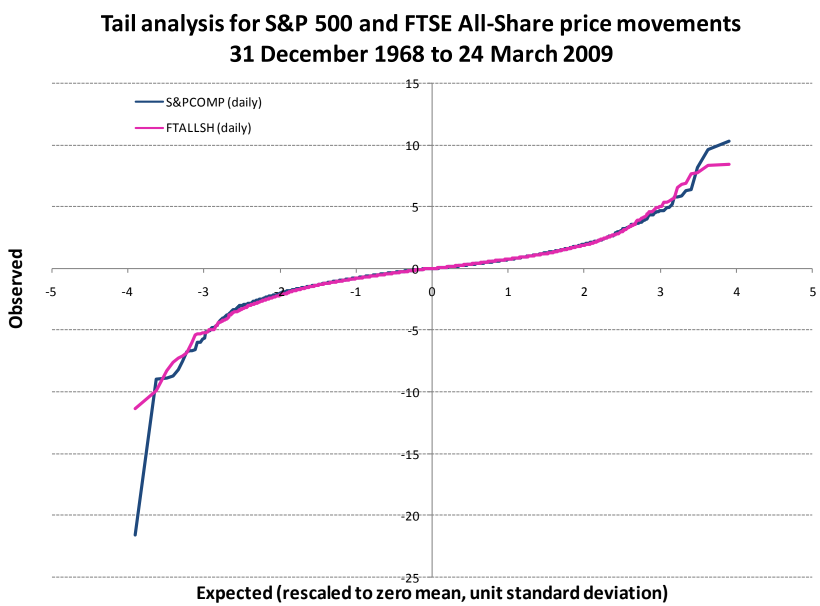
Threadneedle, FTSE

Thomson Datastream

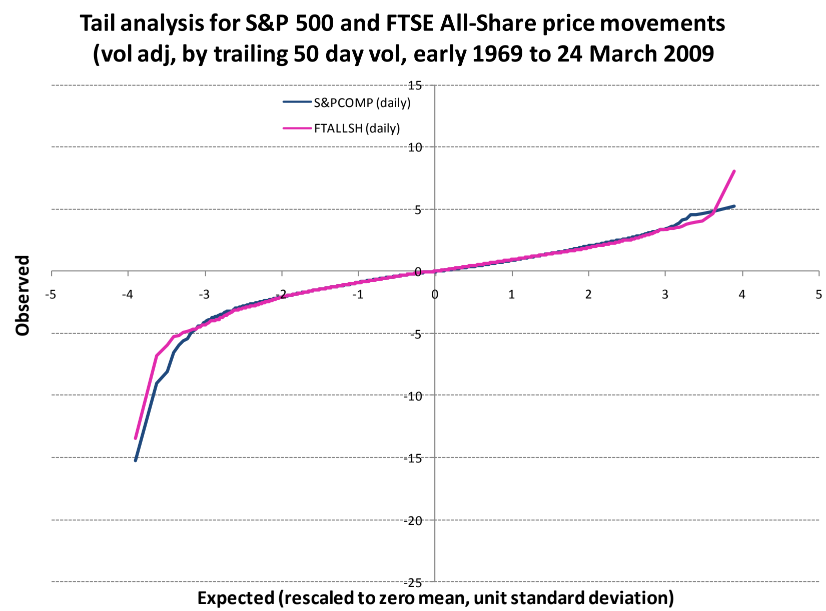
# And over longer time periods



## Raw Data




## With Short-term Volatility Adjustment




Source: Threadneedle, S&P, FTSE, Thomson Datastream

# Crowded trades and leverage



- Some fat tails still seem to come “out of the blue”
  - E.g. Quant funds in August 2007
  - Too many investors in the same *crowded trades*? Behavioural finance implies potentially unstable
  - For less liquid investments, impact may be via an apparent shift in price basis
- System-wide equivalents via leverage?
  - Leverage introduces/magnifies *liquidity* risk, *forced unwind* risk and *variable borrow cost* risk
  - Like selection, involves behavioural finance effects

# Extreme Value Theory (EVT)



- EVT an enticing prospect
  - Appears to offer a mathematically sound way of identifying shape of the ‘tail’ of a distribution, and hence identifying likelihood of extreme (i.e. rare) events
  - Capital adequacy seeks to protect against (we hope) relatively rare events
  - Insurance and credit risk pricing can be dominated by potential magnitude and likelihood of large losses, which are also (we hope) rare
- But bear in mind
  - Inherent unreliability of extrapolation – including extrapolation into the tails of a probability distribution
  - Possibility (indeed probability) that the world is not time stationary
  - We may need to consider a multivariate analogue for portfolio construction

# Restatement of EVT results



- Suppose interested in risk measures relating to losses, $x_j$
- EVT aims to supply two closely related results:

1. Distributions of ‘block maxima’ (or ‘block minima’), i.e. maximum value of $x_i$ in blocks of $m$ observations of $x$ (more traditional use of EVT, wasteful of data):

$$m_j \equiv \max\left\{x_k : (j-1)m+1 \le k \le jm\right\}$$

2. Distributions of ‘threshold exceedances’ (i.e. ‘peaks-over-thresholds’), where $u$ is a predetermined high threshold and we focus on realisations of $x_j$ that exceed $u$ (more relevant to, e.g. computation of Value at Risk i.e. VaR), i.e.:

$$y_i \equiv x_i - u \qquad \text{for } i \text{ s.t. } x_i - u > 0$$

# Main result for block maxima



- Suppose that $x_1$, $x_2$, ..., $x_n$, ... are independent random variables possessing same cumulative distribution function, $F$, and that there exist sequences $a_n$ and $b_n$ such that the following tends in distributional form to $Q$, a non-degenerate probability distribution from which random variable $y$ is drawn

$$\frac{\max\{x_1,\ldots,x_n\}-b_n}{a_n} \xrightarrow{D} y$$

- Then $Q$ is equal to $H(\xi)$ for some $\xi$ (if $a_n$ and $b_n$ appropriately scaled) where $H(\xi)$ is the generalised extreme value (GEV) distribution. $F$ is then said to be in the maximum domain of attraction of $H(\xi)$

| value of $\xi = 1/\alpha$ | GEV sub-type | (cumulative) distribution function |
|---|---|---|
| $\xi = 0$ | Gumbel | $\exp(-\exp(-x))$ for $-\infty < x < \infty$ |
| $\xi > 0$ | Fréchet | $\exp\left(-(1+\xi x)^{-1/\xi}\right)$ for $1+\xi x > 0$, otherwise 0 |
| $\xi < 0$ | Weibull | $\exp\left(-(1-\xi x)^{1/\xi}\right)$ for $1-\xi x > 0$, otherwise 1 |

# Main result for excesses



- Let $F_u$ be defined as follows: $F_u(z) \equiv \Pr\left(x-u<z \middle| x>u\right)$

- Then under same hypotheses as applied to block maxima we have:

$$\lim_{u\to x_f} \sup_{0<z<x_f-u} \left|F_u(z) - G_{\mu,\sigma,\xi}(z)\right| = 0$$

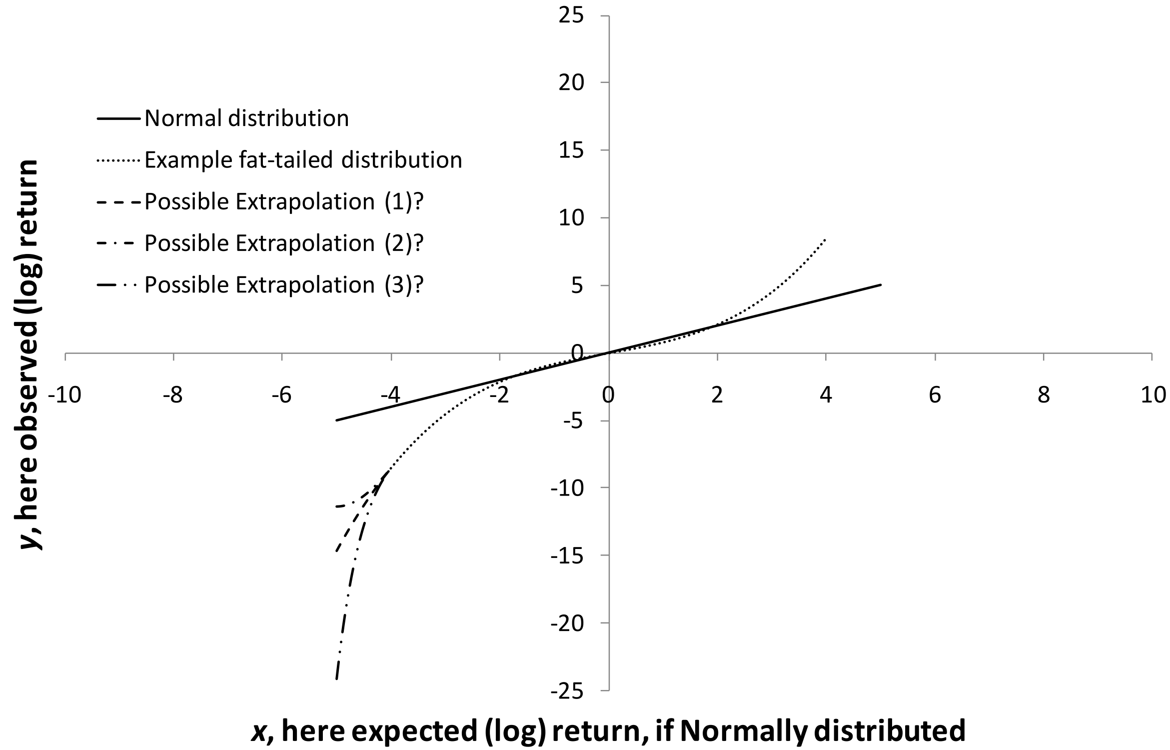
- Where $G_{\mu,\sigma,\xi}(z)$ has the form:

$$G_{\mu,\sigma,\xi}(z) = \begin{cases} 1-\left(1+\xi\dfrac{(z-\mu)}{\sigma}\right)^{-1/\xi}, & \xi \neq 0 \\ 1-\exp\left(-\dfrac{(z-\mu)}{\sigma}\right), & \xi = 0 \end{cases}$$

- Here $\xi$ has the same type of meaning as before, e.g. $G_{\mu,\sigma,\xi}(z)$ is in the maximum domain of attraction of $H(\xi)$

# Potential weaknesses

- EVT seems very helpful
  - Characterises limiting distributions very succinctly
  - But requires regularity conditions that may not be satisfied
  - Relies on existence of a limiting distribution but this is not guaranteed
- At issue is potential unreliability of **extrapolation**
  - E.g. Press et al. (2007)




Source: Nematrian

# Using EVT and alternatives to Estimate VaRs

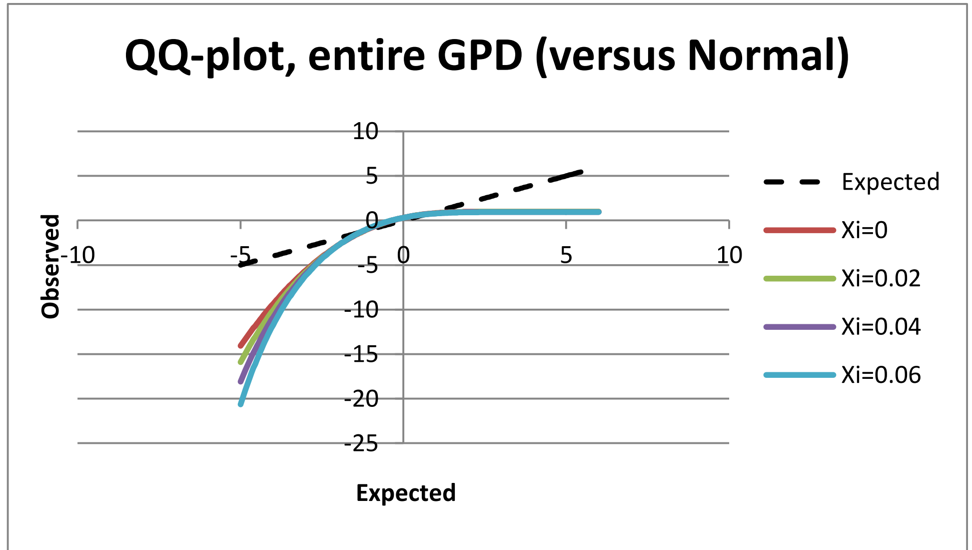


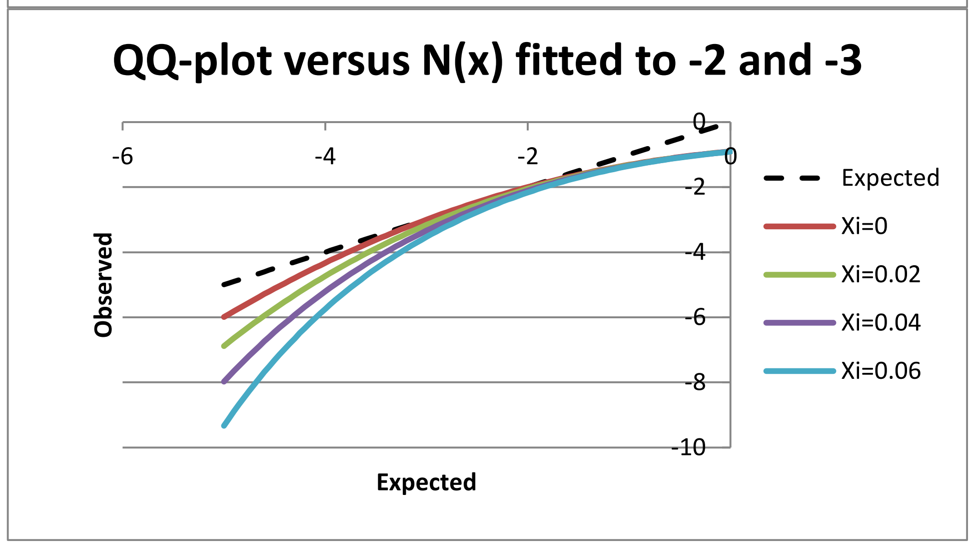
- Assume limiting distribution of tail is fat-tailed GPD
  - Thus use approximation: $F_u\left(y\right) \approx G_{\mu,\sigma,\xi}\left(y\right)$
  - Problem of estimating *F* and its (tail) quantiles then reduces to problem of estimating $\mu$, $\sigma$ and $\xi$ for the approximating generalised Pareto distribution
  - Can be done using mean excess functions, maximum likelihood (ML) estimation, method of moments etc.
- But equally we could fit to the relevant part of the QQ-plot using any other reasonable form of curve fitting approach
  - E.g. polynomial curve fit such as a cubic (see earlier), as long as the resulting extrapolation is credible

# Some subtleties of EVT



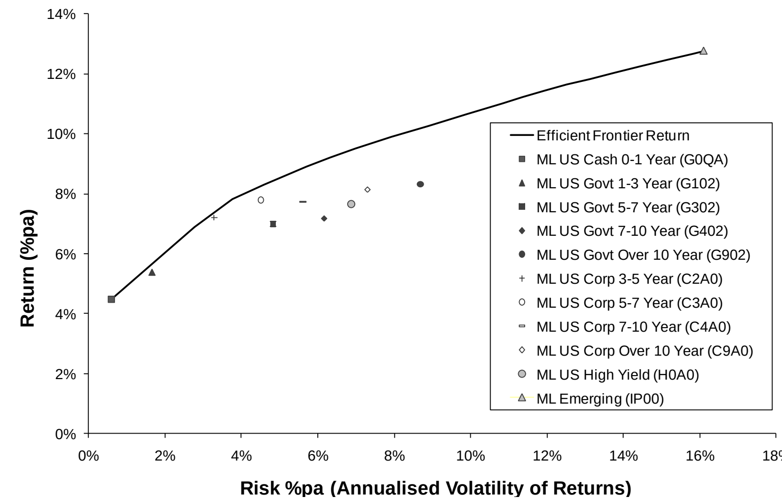
- QQ-plot of GPD is convex upwards (if $\xi \geq 0$)
- If data is Normal then fitting a GPD may overstate size of extreme events somewhat
  - Since Normal has same tail characteristics as $\xi = 0$ GPD
- In real life often have multiple series / loss types
  - Can construct *multivariate* EVT theory, but more complex
  - E.g. McNeil, Frey & Embrechts (2005)

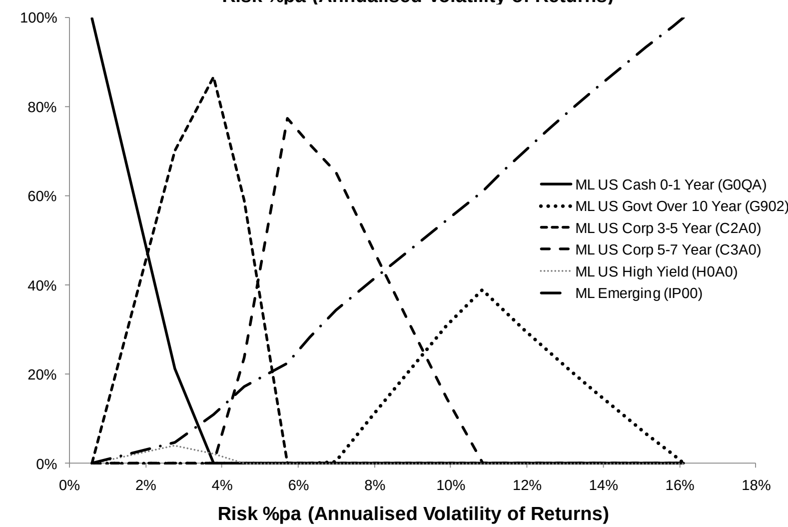


# Portfolio construction

- Traditional (quantitative) approach involves **portfolio optimisation**
  - Typically mean-variance optimisation
  - Identify expected return ('alpha') from each position
  - Maximise expected return for a given level of risk (subject to constraints, e.g. weights sum to unity)
  - Maximise $\mathbf{a}.\mathbf{r} - \lambda.\mathbf{a}^T\mathbf{V}\mathbf{a}$
- Intrinsically **multivariate**

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# Portfolio construction – sensitivities



- Output results are notoriously sensitive to input assumptions
- Possible responses:
  - Treat quant models with scepticism (the fundamental manager’s approach?)
  - Use robust approaches, Bayesian priors/anchors, e.g.
    - Black-Litterman
    - ‘Shrinkage’
    - Position limit ‘priors’ (e.g. 1/N, long-only etc.)
    - Resampling
  - Focus on reverse optimisation

# Incorporating fat tails – Solution A – simplest



- Most important (predictable) single contributor to fat tails seems to be time-varying volatility. So:
  - Calculate covariance matrix between return series after stripping out effect of time-varying volatility
  - Optimise as you think fit (standard, “robust”, Bayesian, BL, ...), using adjusted covariance matrix
  - Adjust risk aversion/risk budget appropriately
  - Then unravel time-varying volatility adjustment
  - Or derive implied alphas using same adjusted covariance matrix
- Implicitly assumes all adjusted return series ‘equally’ fat-tailed

# Incorporating fat tails – Solution B – more sophisticated



- Model with a mixture of multivariate Normal distributions (or GPDs, …)
  - Time-stationary? Maybe not realistic?
  - Time-varying?
    - (Discrete) regime switching, and/or
    - (Continuous) parameterisation (and continuous time?)
- However:
  - Even a mixture of just two multivariate Normal distributions involves estimation of twice as many parameters
  - Results even more sensitive to input assumptions
  - Time varying => dynamic => sensitivity to transaction costs

## Summary



- Fat-tailed behaviour
  - Very common in practice
  - Several intrinsic reasons for its existence, including time-varying world
- Extreme Value Theory (EVT)
  - Enticingly simple (at least in concept)
  - But subject to same underlying issues as any other form of extrapolation
- Portfolio construction can be refined to cater better for extreme events
  - Adjust for (global) time-varying volatility
  - Any further refinements become very complex

# Important Information